14th Plasma Workshop for Young Scientists
Naka Fusion Institute, Japan Atomic Energy Agency

Hall MHD in a Magnetospheric, Weakly Ionized Plasma Device

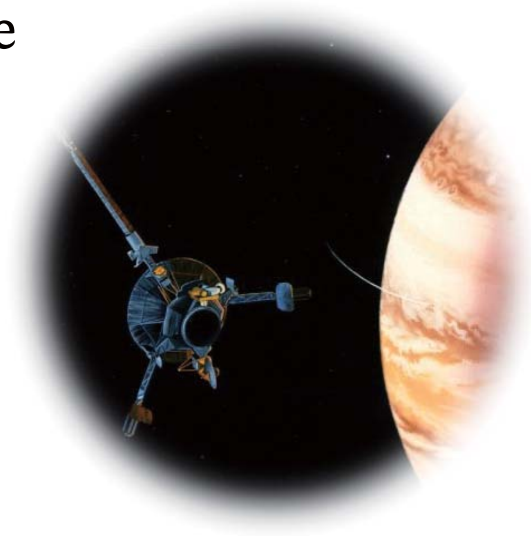
Germán Vogel
Yoshida Laboratory

東京大学
THE UNIVERSITY OF TOKYO

TRANSDISCIPLINARY SCIENCES, GSFS

Motivation

- Hasegawa (1987) inspired in Jovian magnetosphere for fusion confinement which can yield $\beta \gg 1$ free from disruptions.



- Up to now, dipole devices have focused on levitation of a central ring within compact geometry – away from dipole approximation.

- Study of Hall MHD hierarchy scales essential to understand phenomena in weakly ionized plasmas.

A. Hasegawa, Comments Plasma Phys. Control. Fusion 1, 147 (1987).

Objectives

- Design and construction of an experimental device for study of planetary-like magnetospheric plasma configuration at dipole approximation.

- Assessment of Hall MHD regimes and Kolmogorov scales of energy dissipation applicable for weakly ionized plasma, and achieve experimental reading.

Kolmogorov scales
Important to understand the nature of phenomena involved, via the mechanisms of energy dissipation.

Weakly Ionized Plasma

- Electron dynamics, $$0=-\vec{\nabla} p_e - e n_e\left[\vec{E}+\frac{\vec{V}_e\times\vec{B}}{c}\right]-\nu_{en}\rho_e\left(\vec{V}_e-\vec{V}_n\right) \quad (1)$$

$$\left(\alpha=\rho_n/\rho_i\gg 1\right)$$

- Ion dynamics, $$0=-\vec{\nabla} p_i + e n_i\left[\vec{E}+\frac{\vec{V}_i\times\vec{B}}{c}\right]-\nu_{in}\rho_i\left(\vec{V}_i-\vec{V}_n\right) \quad (2)$$

- Neutrals behavior, $$\rho_n\left[\frac{\partial \vec{V}_n}{\partial t}+\left(\vec{V}_n\cdot\vec{\nabla}\right)\vec{V}_n\right]=-\vec{\nabla} p_n-\nu_{ni}\rho_n\left(\vec{V}_n-\vec{V}_i\right)-\rho_n\vec{\nabla}\phi_g+\mu\nabla^2\vec{V}_n \quad (3)$$

- Neutrals under Lorentz, $$\rho_n\left[\frac{\partial \vec{V}_n}{\partial t}+\left(\vec{V}_n\cdot\vec{\nabla}\right)\vec{V}_n\right]=-\nabla p+\frac{\vec{J}\times\vec{B}}{c}-\rho_n\vec{\nabla}\phi+\mu\nabla^2\vec{V}_n \quad (4)$$

on defining $\vec{V}_n-\vec{V}_i=\frac{\nabla(p_i+p_e)}{\nu_{in}\rho_i}-\frac{\vec{J}\times\vec{B}}{c\,\nu_{in}\rho_i}$, $\vec{J}=-e n_e\left(\vec{V}_e-\vec{V}_i\right)$

Krishan and Yoshida, PoP. 13 (2006)

Hall and Ambipolar Diffusion Terms

- In weakly ionized plasma, dissipation mechanisms depend upon (1) plasma properties, (2) energy injection rate.

- Description of weakly ionized plasma dynamics,

$$\frac{\partial \vec{V}}{\partial t}+\left(\vec{V}\cdot\vec{\nabla}\right)\vec{V}=-\nabla h+\frac{\vec{J}\times\vec{B}}{c\,\rho_n}+\mu\nabla^2\vec{V}$$

$$\frac{\partial \vec{B}}{\partial t}=\vec{\nabla}\times\left\{\left[\vec{V}-\frac{\vec{J}}{e\,n_e}+\frac{\vec{J}\times\vec{B}}{c\,\nu_{in}\rho_n}\right]\times\vec{B}\right\}+\eta\nabla^2\vec{B}$$

$$\frac{\partial \vec{V}}{\partial t}+\left(\vec{V}\cdot\vec{\nabla}\right)\vec{V}=-\vec{\nabla}h+\vec{J}\times\vec{B}+\epsilon_\mu\nabla^2\vec{V} \qquad (5)$$

$$\frac{\partial \vec{B}}{\partial t}=\vec{\nabla}\times\left\{\left[\vec{V}-\epsilon_H\vec{J}+\epsilon_A\vec{J}\times\vec{B}\right]\times\vec{B}\right\}+\epsilon_\eta\nabla^2\vec{B} \qquad (6)$$

where $\epsilon_H:=\alpha\frac{c/\omega_{pi}}{L_0}=\alpha\frac{\delta_i}{L_0}$, $\epsilon_\eta:=\eta\frac{t_0}{L_0^2}=\frac{\eta}{L_0V_0}$, $\epsilon_A:=\epsilon_H\frac{\omega_{ci}}{\nu_{in}}$, $\epsilon_\mu:=\mu\frac{t_0}{L_0^2}=\frac{\mu}{L_0V_0}$

Krishan and Yoshida, Mon. Not. R. Astron. Soc. 395, (2009).

Reynolds Number and Kolmogorov Scales

- Reynolds number as ratio of advective and dissipative terms.

example (viscosity dissipation)

$$\frac{\partial \vec{V}}{\partial t}+\left(\vec{V}\cdot\vec{\nabla}\right)\vec{V}=-\vec{\nabla}h+\vec{J}\times\vec{B}+\epsilon_\mu\nabla^2\vec{V}$$

$$R_\mu(K)=\frac{\left(\vec{V}\cdot\vec{\nabla}\right)\vec{V}}{\epsilon_\mu\nabla^2\vec{V}} \Rightarrow R_\mu(K)\approx\frac{V_K}{\epsilon_\mu K}$$

$$K_\mu=\frac{V_{K_\mu}}{\epsilon_\mu}=\xi^{3/4}\epsilon_\mu^{-3/4}$$

$\xi=\epsilon_\mu K_\mu^2 V_\mu^2$: energy injection rate

Reynolds Number and Kolmogorov Scales

- Reynolds number as ratio of advective and dissipative terms.

- When energy injection rate is specified, and plasma parameters defined, we can determine Kolmogorov scales respective to each dissipation mechanism,

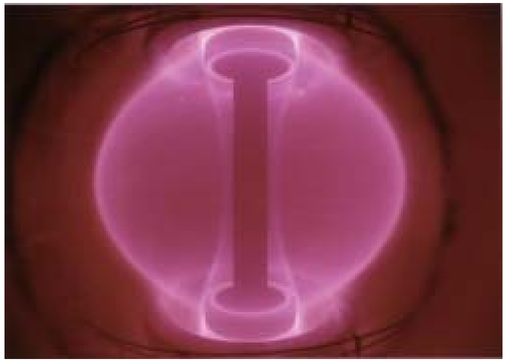
Dissipation mechanism	MHD regime ($\epsilon_H K < 1$)	Hall MHD regime ($\epsilon_H K > 1$)
Viscosity	$K_\mu = \xi^{1/4} \epsilon_\mu^{-3/4}$	$K_{\mu H} = (\epsilon_H^{-1})^{1-q} K_\mu^q (< K_\mu)$
Resistivity	$K_\eta = \xi^{1/4} \epsilon_\eta^{-3/4}$	$K_{\eta H} = \epsilon_H K_\eta^2 (> K_\eta)$
Ambipolar	$K_A = \xi^{1/2} \epsilon_A^{-3/2}$	$K_{AH} = \epsilon_H^{-2} K_A^{-1} (< K_A)$

Table 1. Energy dissipation scales

Krishan and Yoshida, Mon. Not. R. Astron. Soc. 395, (2009).

Dipole Confinement Approach

Tokamak, Spheromak



$\beta \sim 1$

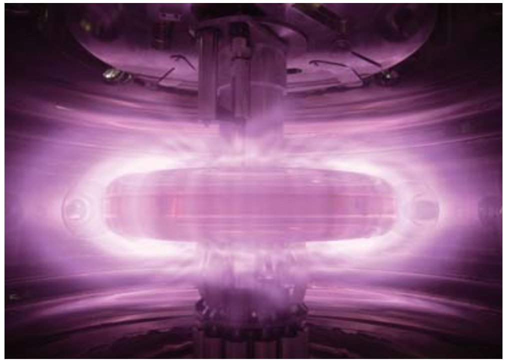
negative NC effects

divertor heat load

tritium blanket

non steady state

Dipole



$\beta > 1$

no NC effects

large flux expansion

advanced fuels

steady state

A new experimental device

- No levitating coil eases use and allows continuous operation.
- Supported magnet, it is a simpler task to generate E field in order to induce plasma flow.
- Dipole approximation can be examined and provide conclusions for planetary magnetospheres.
- Examine neutrals under Lorentz force using low-ionized plasma.
- May also serve to contrast Hasegawa's predictions vs RT-1 empirical observations.

Experimental Setup

- Nd permanent magnet mounted on a steel rod, at half plane of 2.45 GHz ECH microwave port.
- To allow large flux expansion, $B \propto r^{-3}$
- $r_M / r_{vac} \sim 10^2$, $h_M / h_{vac} \sim 10^2$
- $B_M = 0.5T$

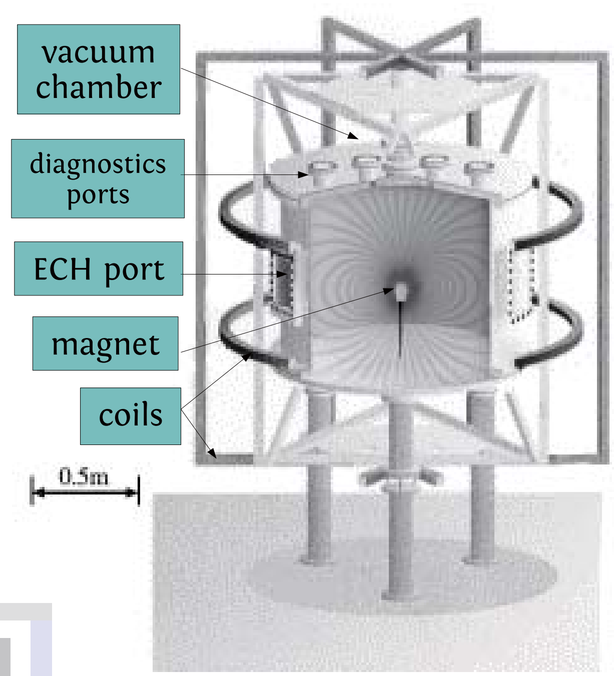


Figure 1. Experimental device.

	Br (T)	Hci (kA/m)	BH (kJ/m³)	Tc (°C)
Ferrite	0.2-0.4	100-300	10-40	450
SmCo	0.8-1.1	600-2000	150-200	250-700
NdFeB	1.0-1.4	750-2000	300-400	200-400

Table 2. Rare-earth and ferrite magnets.

Magnetic Field Calculations

- Magnet approximated as solenoid,

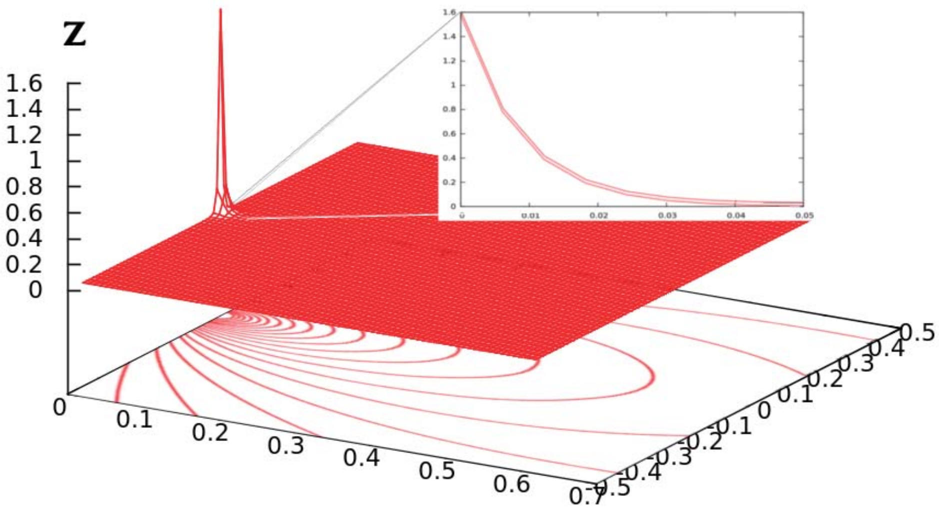
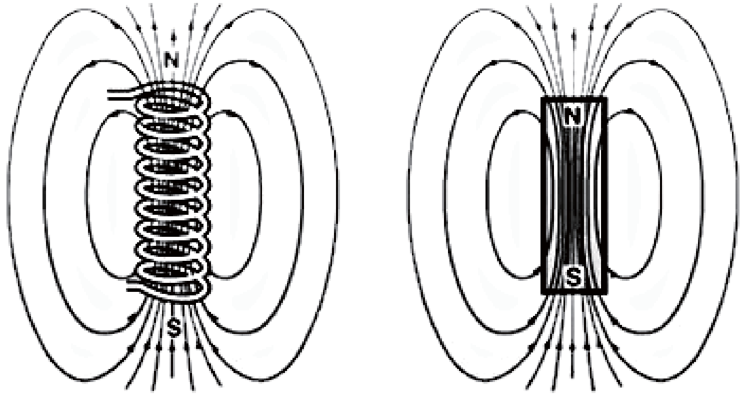
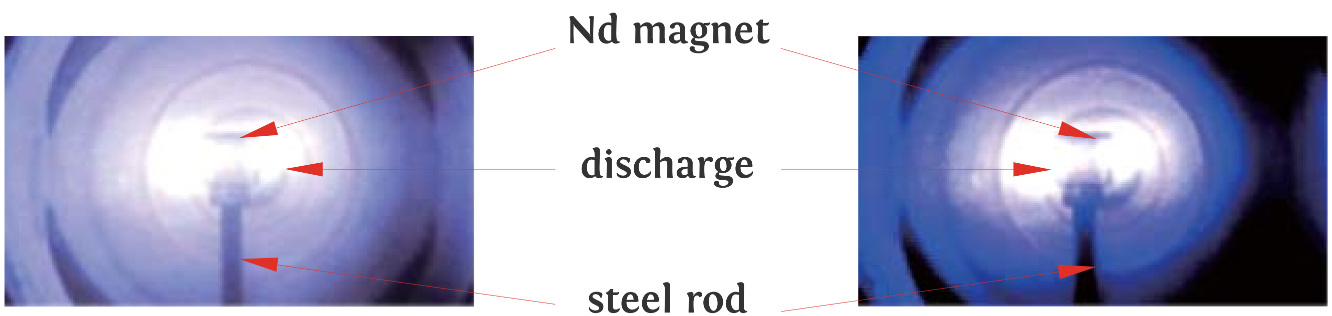


Figure 2. Magnetic calculations



$$B_r=\frac{\mu_0 I}{2\pi}\frac{z}{r\sqrt{(R+r)^2+z^2}}\left[-K+\frac{R^2+r^2+z^2}{(R-r)^2+z^2}E\right]$$

TEST CHAMBER



Setting Up & Reachable Pressure

- Mean free paths for main molecules,

$$\lambda_{N_2}(p)=\frac{7.0\times10^{-5}}{p}m \quad , \quad \lambda_{H_2O}(p)=\frac{4.7\times10^{-5}}{p}m$$

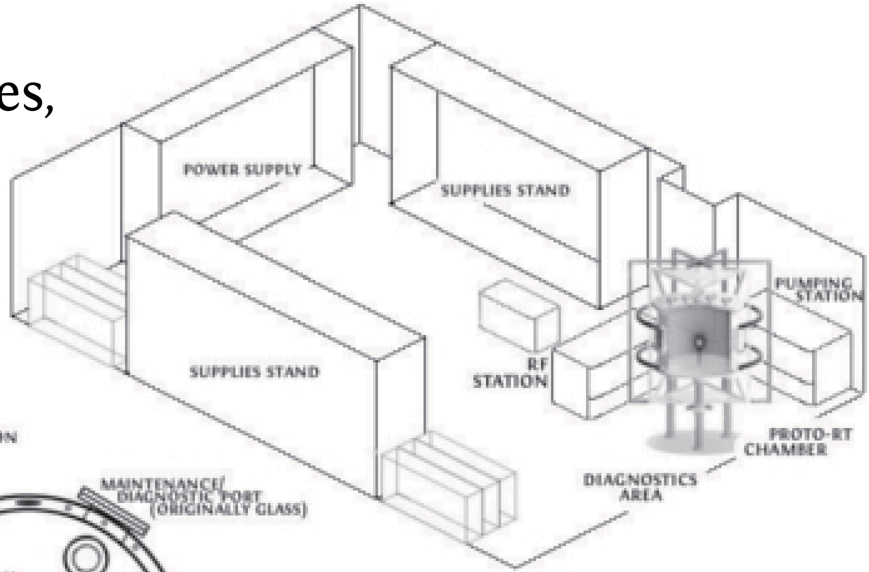
$$\lambda_{O_2}(p)=\frac{7.6\times10^{-5}}{p}m$$

- Necessity of $\lambda_{H_2O}<\lambda_{H_2}$

  translates into $p\leqslant10^{-6}\,Torr$

- 500 l/s turbomolecular pump,
  to find effective pumping speed,

$$\frac{1}{S_{eff}}=\frac{1}{S_p}+\frac{1}{C} \qquad p=\frac{Q}{S_{eff}}=1.4\times10^{-7}\,Torr$$



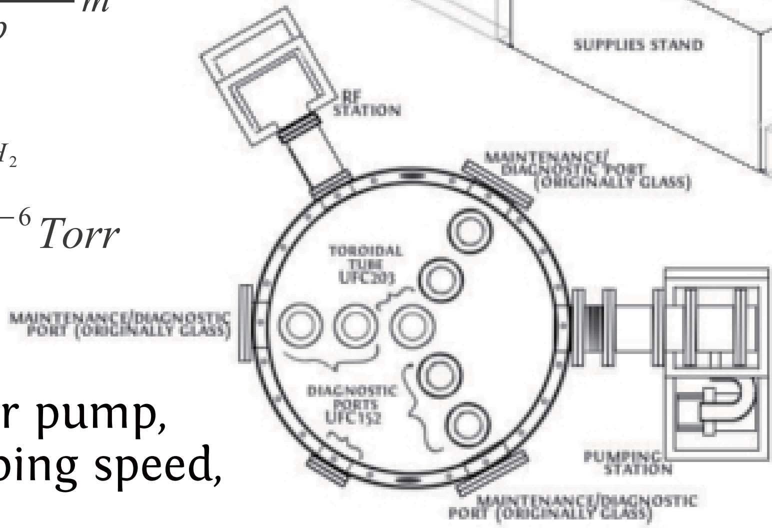


Figure 3. Chamber and set up

Foundations of Vacuum Science, Lafferty (1998)

Expectations

- Over distance, ρ_i/ρ_n via $$\frac{n_i}{n_n}=2.4\times10^{15}T^{3/2}\frac{n_{Na}}{n_i n_n}\exp\left(-U_{Na}/K_B T\right) \qquad (7)$$

- Over energy injection rates, $K_A/K_{\eta,\mu}$ and K_η/K_μ

- Choosing parameters at normalization distance where to estimate injection rate.

EXAMPLE
SOLAR
ATMOSPHERE

(Krishan, Yoshida)

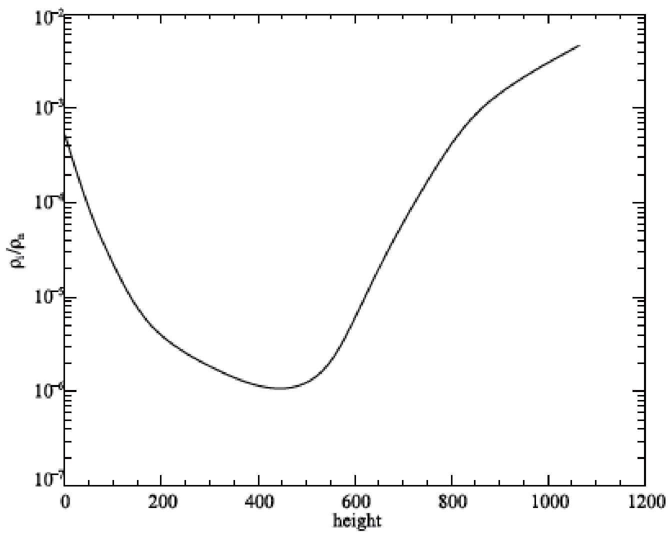


Figure 4. Ionization fraction over height

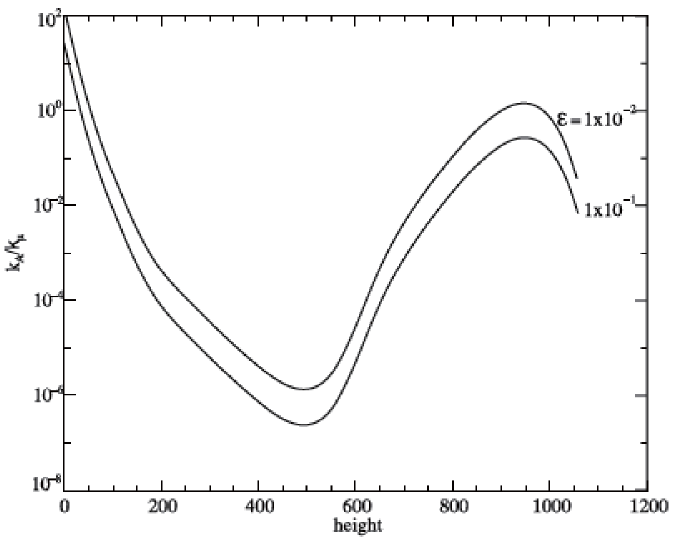


Figure 5. Kolmogorov comparison

Summary and Conclusions

- A new device expects to confine plasma testing the dipole approximation and Hasegawa requirements.
- Inspiration in magnetospheric empirical observations of natural high-β confinement.
- Looking for experimental confirmation of expected Kolmogorov scales and profiles.
- Towards the implementation of diagnostics and electric field.