The effect of magnetic shaping on zonal flow damping in a toroidal global gyrokinetic simulation

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Global gyrokinetic (GK) global simulation is considered to be an essential tool to understand micro-scale instability and associated turbulent transport phenomena including the profile stiffness / resilience and transport barrier formation. While many GK codes already exist, most of them rely on constraining hypotheses such as a circular section (despite the D-shape of actual tokamak such as ITER) to simplify the equation systems.

Our 5D full-f toroidal GK Vlasov simulation code, GKNET, has been upgraded, with the addition to its real space field solver of a new high accuracy ZF solver, based on a diagonalisation of the ZF equation. In addition to being more rigorous near the center of the poloidal plane compared to those local approximations, a solver based, this method allows for accurate results on low resolution grids. This upgraded code was used to study GAM damping in elliptic and both positive and negative D-shaped configurations. While the influence of elongation had been partially studied, we introduce new results on the influence of triangularity on the damping rate, showing an assymmetry allowing negative triangularities to damp the ZF faster.

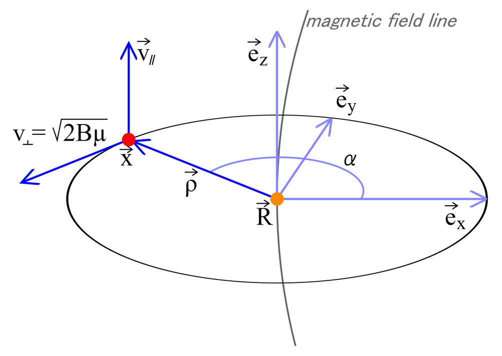
Gyrokinetics formalism

The Vlasov-Poisson system is expressed as:

$$\begin{cases}\frac{\partial f}{\partial t}+\vec{v}\cdot\frac{\partial f}{\partial \vec{x}}+\frac{q}{m}(-\nabla\Phi+\vec{v}\times\vec{B})\cdot\frac{\partial f}{\partial \vec{v}}=0 & \text{(Vlasov equation)}\\ \Delta\Phi=-\frac{q_e}{\epsilon_0}\int(F_i-F_e)d^3v & \text{(Poisson equation)}\end{cases}$$

where f is the distribution function describing the plasma's configuration in the 6-dimensional phase space and Φ is the electric potential.

The particles' helicoidal trajectories around the magnetic field lines can be described through the position of the center of this gyration $\vec{R}$, the parallel speed $v_{/\!/}$, the magnetic moment $\mu=v_\perp^2/(2B)$ that defines gyration speed and radius and the angle α of the gyration:



$$(\vec{x},v_x,v_y,v_z)\mapsto(\vec{R},v_{/\!/},\mu,\alpha)$$

As the gyration radius is very small compared to the characteristic scale of the equilibrium structures, the variables can be averaged in α (over these circles) reducing the number of dimensions to 5. However, this transformation yields in the Poisson equation such gyro-averaged terms whose computation can be difficult.

Derivation of the model

The gyrokinetic Vlasov equation which describes the evolution of the guiding center distribution f_s of the specie concerned is derived using Hamiltonian mechanics as:

$$\frac{\partial f_s}{\partial t}+\dot{\boldsymbol{R}}\cdot\frac{\partial f_s}{\partial \boldsymbol{R}}+\dot{v}_{/\!/}\frac{\partial f_s}{\partial v_{/\!/}}=0 \quad\text{where}\quad \begin{cases}\dot{\boldsymbol{R}}=v_{/\!/}\boldsymbol{b}+\frac{\boldsymbol{b}}{m_sB^*_{/\!/}}\times\left(e_s\boldsymbol{\nabla}\langle\Phi\rangle_{\boldsymbol{x}}+\frac{m_sv_{/\!/}^2+\mu B}{B}\boldsymbol{\nabla}B+\frac{v_{/\!/}^2}{B}(\boldsymbol{\nabla}\times\boldsymbol{B})\times\boldsymbol{b}\right)\\ \dot{v}_{/\!/}=\frac{-1}{m_sB^*_{/\!/}}\boldsymbol{B^*}\cdot(e_s\boldsymbol{\nabla}\langle\Phi\rangle_{\boldsymbol{x}}+\mu\boldsymbol{\nabla}B)\end{cases} \qquad (1)$$

The electrostatic potential Φ and is given by the GK quasi-neutrality condition reads:

$$\frac{e_i}{T_i}(\Phi-\langle\!\langle\Phi\rangle\!\rangle)+\frac{e}{T_e}(\Phi-\overline{\Phi})=\frac{m_i^2}{n_0}\int B^*\langle f_i-f_0\rangle_{\boldsymbol{R}}\,dv_{/\!/}d\mu \qquad (2)$$

and where $\langle\cdot\rangle_{\boldsymbol{x}}$ and $\langle\cdot\rangle_{\boldsymbol{R}}$ denotes the simple gyro-averaging (depending on the variable's initial configuration space) and the term $\langle\!\langle\Phi\rangle\!\rangle$ is the double averaging, defnied as:

$$\langle\!\langle\Phi\rangle\!\rangle(\boldsymbol{x})=\frac{1}{2\pi}\int\langle\Phi\rangle_{\boldsymbol{x}}f_M\delta(\boldsymbol{R}+\boldsymbol{\rho}-\boldsymbol{x})d\boldsymbol{R}d\alpha d\mu$$

with f_M a Maxwellian in $v_\perp$, slowly varying in $\boldsymbol{R}$ such f_0's dependence is of the form $f_0(\boldsymbol{R},0,v_{/\!/})f_M(\boldsymbol{R},\mu)$ (*i.e.* a Gaussian distribution of variance the thermal velocity v_{th}).

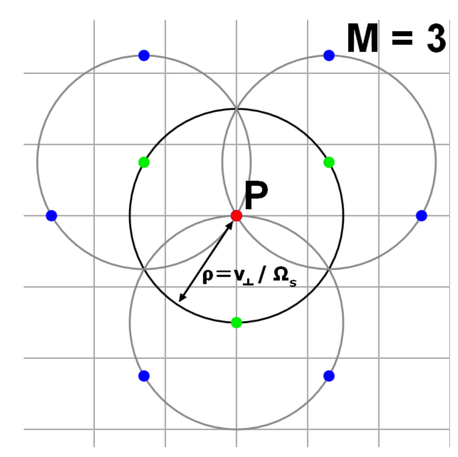
The flux averaged term $\overline{\Phi}$ plays an important role in the damping of zonal flow. The Zonal Flow equation is obtained by computing the flux average of the field equation eq. 2:

$$\overline{\Phi}-\overline{\langle\!\langle\Phi\rangle\!\rangle}=\frac{T_im_i^2}{e_i\,n_0}\overline{\int B^*\langle f_i-f_0\rangle\,dv_{/\!/}d\mu}. \qquad (3)$$

Real space gyro-averagings

Simple and double averagings in real space

Rather than a theoretical simplification of its expression, the simple averaging is computed by sampling a given number of points M on circular orbits (black circle and green points on thetop figure on the right).



The double averaging being the composition of 2 simple averagings, the averaging is here performed over M^2 "secondary points" sampled on "secondary" circular orbits centered on each "primary point" (the blue and red points on the grey circles on the same figure). The integration over $v_\perp$ (*i.e.* in radius), is computed as a weighted sum of double averagings for given radii, the weights and radii being computed numerically to minimise the error.

The error of the 2 or 3-D interpolations and of the sampling on the circles can be estimated theoretically. In the latter case, for a given mode $k_\perp$ in Catesian coordinates, using M points will yield an error of the order of $2J_{2M}(k_\perp v_\perp)/J_0(k_\perp v_\perp)$ for M odd and $2J_M(k_\perp v_\perp)/J_0(k_\perp v_\perp)$ for M even.

Resolution of the ZF equation by diagonalisation

To study the ZF equation eq. 3, expanding an arbitrary flux function Ψ around the magnetic axis, we can derive the parametrisation of the D-shaped magnetic flux surfaces:

$$\begin{cases}R=R_0+\alpha\cos(\theta)-\frac{\alpha^2}{a_0}(\Delta+\delta\sin(\theta)^2) & +O(\epsilon^3)\\ Z=\alpha\kappa\sin(\theta) & +O(\epsilon^3)\end{cases} \qquad (4)$$

where κ is the elongation, δ is the triangularity, Δ is the Shafranov shift and α a linear labelling of flux surfaces (simply equivalent to the radius in circular cases) ranging from 0 to a_0. Using these coordinates, we can establish the equations verified by eigenfunctions:

$$\lambda f_\lambda(\alpha)=\overline{\langle\!\langle f_\lambda\rangle\!\rangle}(\alpha)=\frac{1}{S_\alpha}\iint R\alpha\langle\!\langle f_\lambda\rangle\!\rangle d\theta d\varphi \quad\Rightarrow\quad (1+G_{1,2}\alpha^2)f_\lambda''+\frac{1+G_{2,2}\alpha^2}{\alpha}f_\lambda'+\omega_\lambda^2f_\lambda=0 \qquad (5)$$

where the coefficients $G_{1,2}$ and $G_{2,2}$ are $O((\epsilon+\delta+\Delta)^2)$. The neglection of the first order derivative leads to a usual Fourier solution but is inaccurate near the center of the poloidal section. The solutions read:

$$f_\lambda(\alpha)\propto(1-C_2\alpha^2)J_0(\omega_\lambda\alpha)+\frac{\alpha}{\omega_{\lambda,0}}(2C_2+C_1\alpha^2)J_1(\omega_\lambda\alpha) \qquad \begin{cases}C_1=G_{1,2}/6\\ C_2=(3G_{2,2}-5G_{1,2})/12\end{cases} \qquad (6)$$

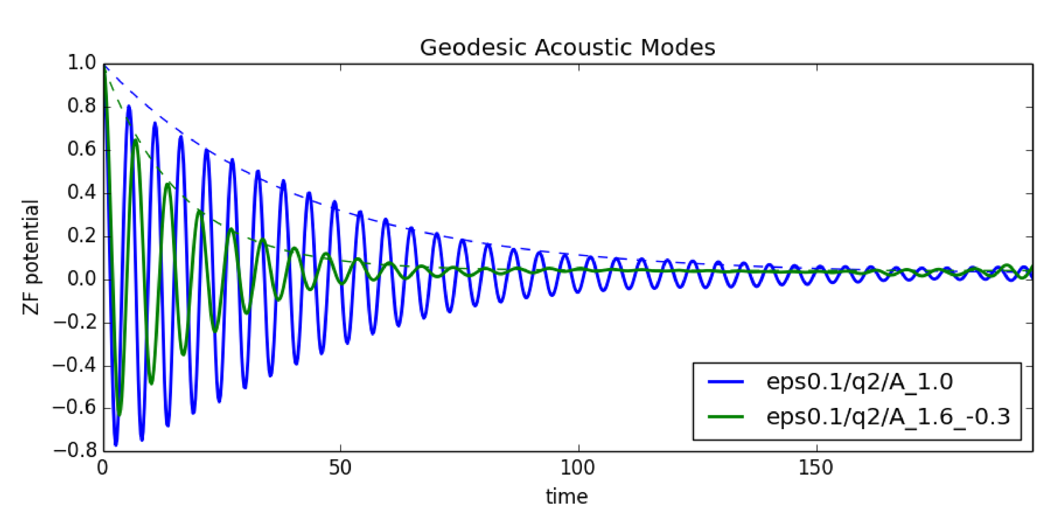
along with the eigenvalues $\lambda_k\simeq\frac{a_0^2}{j_{0,k}^2}\frac{1+\kappa^2}{2\kappa^2}$ for the ZF equation eq. 3, where $j_{0,k}$ being the k-th zero of J_0. Using this result, the ZF equation is fianlly solved by projecting the RHS onto an eigenbasis:

$$\overline{\Phi}-\overline{\langle\!\langle\Phi\rangle\!\rangle}=\sum c_kf_{\lambda_k} \quad\Rightarrow\quad \overline{\Phi}=\sum c_k\lambda_kf_{\lambda_k} \qquad (7)$$

As the eingenvalue decrease very rapidly, eigenbases can be restricted to the first few eigenfunctions, resulting in very small linear equation systems to solve.

Influence of the elongation and triangularity on the ZF damping

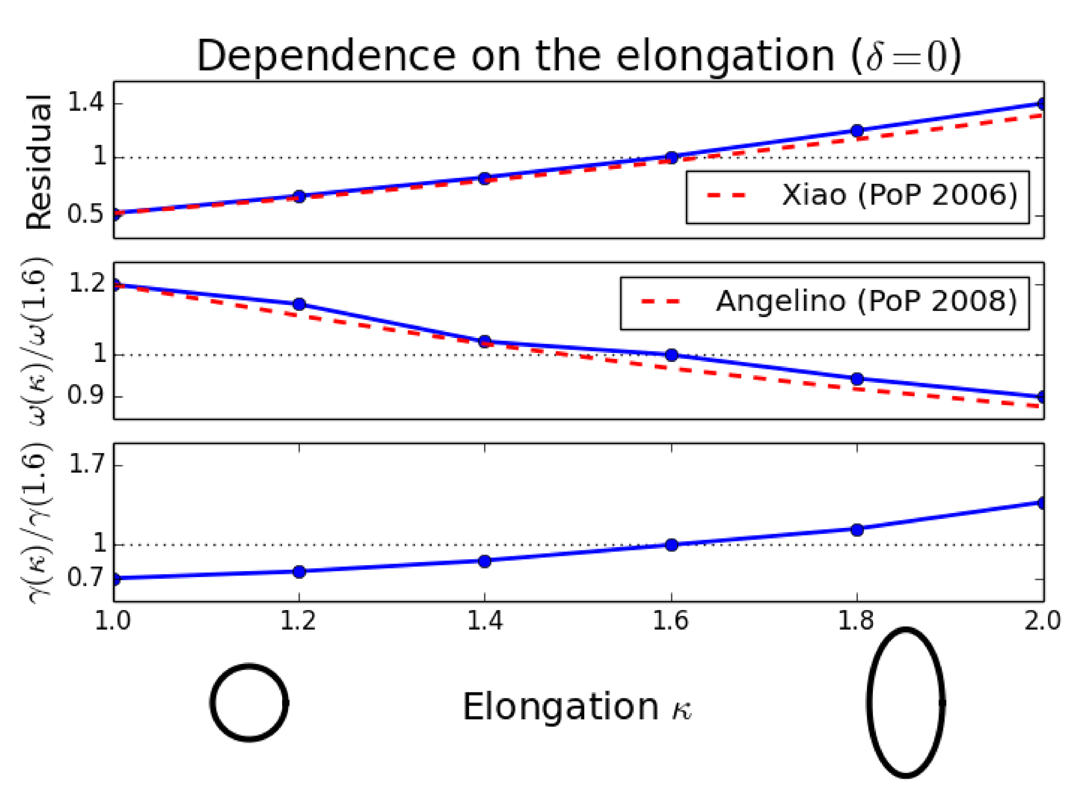
Collisionless ZF damping simulations are performed in a homogeneous plasma with $L_n=L_T=\infty$ in which we follow the evolution of the amplitude of an initial pertubation of the form $\delta f=10^{-3}\sin\left(\frac{\pi}{2}(1-r/a)\right)$. The 3 parameters of interest are the residual ZF, the oscillation frequency ω and the damping rate γ (see figure below).



Time evolution of the ZF for $q=2$ for the circular case (blue) and the negative D-shape case $\kappa=1.6$, $\delta=-0.3$ (green).

While the circular case has been extensively studied, it is not the case of the elliptic or D-shaped case, either theoretically or numerically.

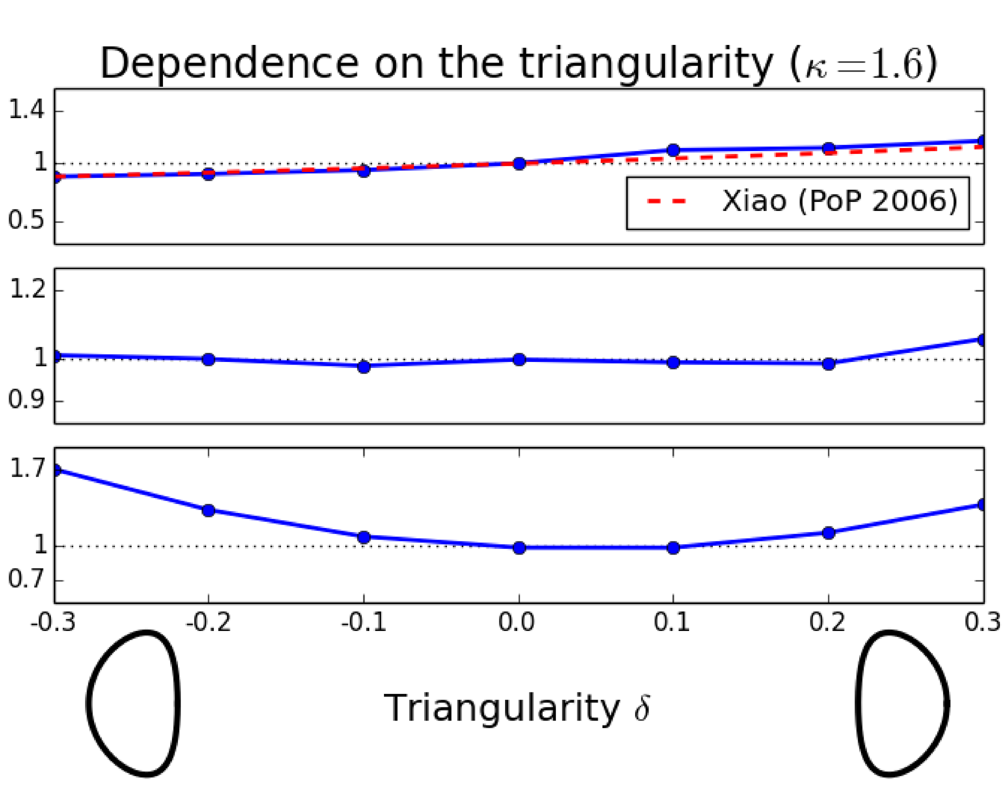
The simulations presented on the right have been performed for $\epsilon=0.1$ and $q=2$ on a grid $(N_R,\ N_Z,\ N_\varphi,\ N_{v_{/\!/}},\ N_\mu)=(64/\kappa,\ 64,\ 1,\ 128,\ 8)$.



Elongation κ is found to enhance the convergence while increasing the limit.

These results mostly corroborate the theoretical result on the residual level by Y. Xiao (Phys. Plasmas, **13**, 082307 (2006)) and numerical ones by P. Angelino (Phys. Plasmas,**15**, 062306 (2008)).

In addition, a theoretical paper by Z. Gao (Phys. Plasmas, **17**, 092503 (2010)) also suggests that the damping rate should increase with elongation, although the formula proposed can only be used for tendencies and not for numerical comparaison.



Triangularity is found to be of little effect on the residual ZF and oscillation frequency. Here again, the influence of triangularity is found to match the theoretical results by Y. Xiao.

However tiangularity seems to strongly enhance ZF damping, with an assymetry favouring negative triangularities.

Summary and conclusion

Our full-f toroidal GK Vlasov code, GKNET, has been upgraded, by introducing to its real space field solver of a new ZF solver which can accurately solve the ZF equation based on a diagonalisation of the ZF equation. This new solver allows for accurate numerical results on very low resolution grids.

With this new solver, GAM damping tests were performed to study the influence of the shape of the magnetic field on the residual ZF level, oscillation frequency and damping rate. Results on the influence of the elongation κ of the section confirm numerical and theoretical works results found in the literature. Additionally, while the triangularity is found to be of litle influence on the residual ZF and oscillation frequency, it is found to strongly enhance the damping of the ZF, with an assymmetry favouring negative triangularities over positive ones.

Future plans

As the ZF soler was developed with the aim of studying GAM damping, it requires a constant temperature over the section while this condition was not necessary in the pre-existing code. This study will be continued to attempt to generalise the diagonalisation of the ZF equation and if possible towards theoretical formulas for the GAM damping parameters based on this approach.

Later GKNET will also be used to study the shaping effects on linear ITG/TEM growth rates and in particular the effects of negative-D shapes.