Two-fluid and FLR effects on MHD instabilities in finite beta plasmas

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- Small-scale effects on RT instability
  - ➢ Local analysis in the short wavelength limit

    Two-fluid and Finite Larmor Radius (FLR) stabilization of RT instability

    — K.V Roberts and J.B. Taylor, PRL **8**, 197 (1962)
      - Low-beta, isothermal
      - Complete stabilization due to ion FLR and Hall effect for short wavelength perturbation

    — P. Zhu, D.D. Schnack *et al.*, PRL **101**, 085005 (2008)
      - Absence of complete FLR stabilization for finite beta plasma with non-uniform temperature
      - Confirmed the extended-MHD simulation results for fusion plasmas
  - ➢ IDG (ion density gradient) mode [P.W. Xi *et al.*, Nucl. Fusion **53**, 113020 (2013)]
      - Finite beta
      - Unstable mode appears due to density gradient in two-fluid model
      - Completely stabilized by adding gyroviscosity

- Goto, Miura, Ito, Sato and Hatori [PFR **9**, 140376 (2014), PoP **22**, 032115 (2015)]
    - RT (interchange g mode), FLR or two fluid, finite beta, non-const. *T*, non-uniform magnetic field
    - Strong stabilization occurs when both of FLR and two-fluid effects are included.
    - Stability analysis for more general conditions is needed for comparison with extended MHD simulation results.

- Tearing mode instability in two-fluid MHD model
    - Drift tearing instability
        - Ion FLR effects on tearing mode instability [B. Coppi, Phys. Fluids **7**, 1501 (1964)]
        - Gyroviscosity is added to two-fluid MHD
        - Rotation of magnetic islands due to diamagnetic drifts in fusion plasmas was observed.
    - Contributions of heat flux cannot be neglected at low collisionality
        - We have derived eigenmode equations for tearing instability in slab geometry including effects of parallel heat flux in the gyroviscous tensor.
    - Benchmark test with theory of two-fluid tearing mode
        - Slab [Ahedo and Ramos (2009)] and cylindrical [Ramos, APS-DPP 2013] equilibrium

Extended MHD equations for RT instability

- Extended MHD equations

  [P. Zhu, D. D. Schnack *et al.*, PRL **101**, 085005 (2008)]

  Ion gyroviscosity, Hall current and electron pressure are added into MHD equations.

$$\frac{\partial \overline{n}}{\partial \overline{t}} + \overline{\nabla} \cdot (\overline{n}\overline{\mathbf{v}}) = 0,$$

$$\overline{n}\left(\frac{\partial \overline{\mathbf{v}}}{\partial \overline{t}} + \overline{\mathbf{v}} \cdot \overline{\nabla}\overline{\mathbf{v}}\right) = \left(\overline{\nabla} \times \overline{\mathbf{B}}\right) \times \overline{\mathbf{B}} - \overline{\nabla}\overline{p} - \delta\left(\frac{d_i}{L}\right)\nabla \cdot \overline{\Pi}^{gv} + \overline{n}\mathbf{g},$$

$$\overline{\mathbf{E}} + \overline{\mathbf{v}} \times \overline{\mathbf{B}} - \frac{\varepsilon}{\overline{n}}\left(\frac{d_i}{L}\right)\left[\left(\overline{\nabla} \times \overline{\mathbf{B}}\right) \times \overline{\mathbf{B}} - \overline{\nabla}\overline{p}_e\right] = 0,$$

$$\frac{\partial \overline{\mathbf{B}}}{\partial \overline{t}} + \overline{\nabla} \times \overline{\mathbf{E}} = 0, \quad \frac{\partial \overline{p}}{\partial \overline{t}} + \overline{\mathbf{v}} \cdot \overline{\nabla}\overline{p} + \gamma \overline{p}\overline{\nabla} \cdot \overline{\mathbf{v}} = 0, \qquad p_i / p = \tau$$

δ: FLR effect, ε: two-fluid effect

$$\mathbf{B} = B_* \overline{\mathbf{B}}, \quad n = n_* \overline{n}, \quad p = m n_* V_A^2 \overline{p}, \quad \mathbf{v} = V_A \overline{\mathbf{v}}, \quad x = L\overline{x}, \quad g = \left(V_A^2 / L\right) \overline{g}$$

$$V_A = \frac{B_*}{\sqrt{\mu_0 n_* m_i}} \quad \text{(Alfven velocity)} \qquad d_i = \sqrt{\frac{m_i}{\mu_0 n_* e^2}} \quad \text{(ion skin depth)}$$

- Ion FLR effect（$\delta=1$）: gyroviscosity $\overline{\Pi}^{gv}$

$$\overline{\Pi}_{xx}^{gv} = -\overline{\Pi}_{yy}^{gv} = -\frac{\overline{p}_i}{2\overline{B}} \left(\frac{\partial \overline{v}_y}{\partial \overline{x}} + \frac{\partial \overline{v}_x}{\partial \overline{y}} \right), \quad \overline{\Pi}_{xy}^{gv} = \overline{\Pi}_{yx}^{gv} = \frac{\overline{p}_i}{2\overline{B}} \left(\frac{\partial \overline{v}_x}{\partial \overline{x}} - \frac{\partial \overline{v}_y}{\partial \overline{y}} \right),$$

- Two-fluid effect（$\varepsilon=1$）: Hall current and electron pressure

- Linear analysis
  - Perturbation:

$$f_1 = f_1(x) \exp(iky - i\omega t), \qquad v_{z1} = 0, \quad \partial / \partial z = 0$$

  - Linear eigenmode equation:

$$v_{x1}'''' + A(x;\omega,k) v_{x1}''' + B(x;\omega,k) v_{x1}'' + C(x;\omega,k) v_{x1}' + D(x;\omega,k) v_{x1} = 0$$

  - Local (WKB) approximation

$$k \gg d / dx$$

Local dispersion relation at $x=0$

$$D(0;\omega,k) = 0$$

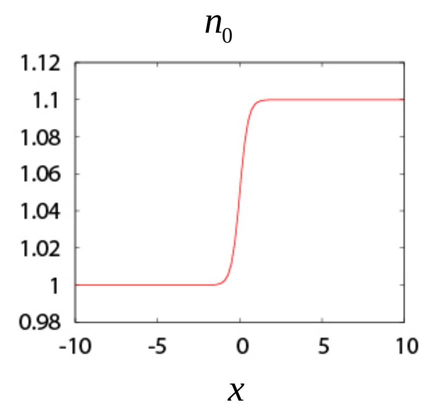
Equilibrium

$$\frac{d}{dx}\left(p_0+\frac{B_0^2}{2}\right)=n_0 g$$

$$p_0+\frac{B_0^2}{2}=P_0$$

$$n_0=\frac{n_1+n_2}{2}+\frac{n_2-n_1}{2}\tanh\left(\frac{2x}{x_2-x_1}\right)$$

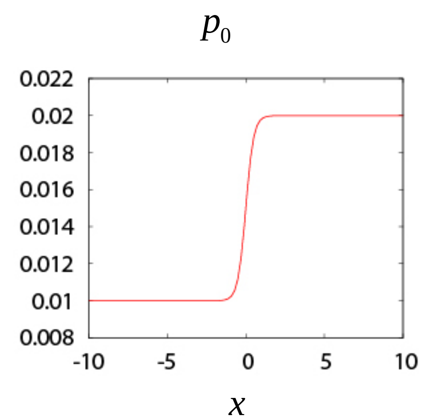
$$P_0=\frac{B_Z^2}{2}+g\left\{\frac{n_1+n_2}{2}x+\frac{(n_2-n_1)(x_2-x_1)}{4}\log\left[\cosh\left(\frac{2x}{x_2-x_1}\right)\right]\right\}$$

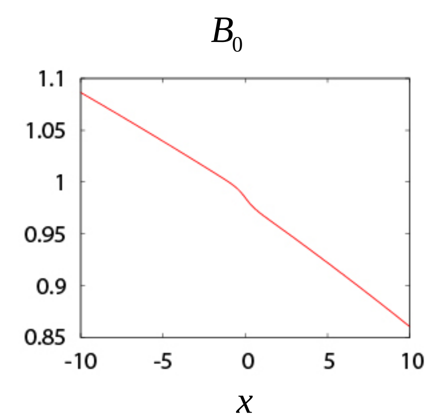


$$p_0=\beta\left[\frac{p_1+p_2}{2}+\frac{p_2-p_1}{2}\tanh\left(\frac{2x}{x_2-x_1}\right)\right],$$

$$B_0=\sqrt{2(P_0-p_0)}$$

– Non-uniform magnetic field

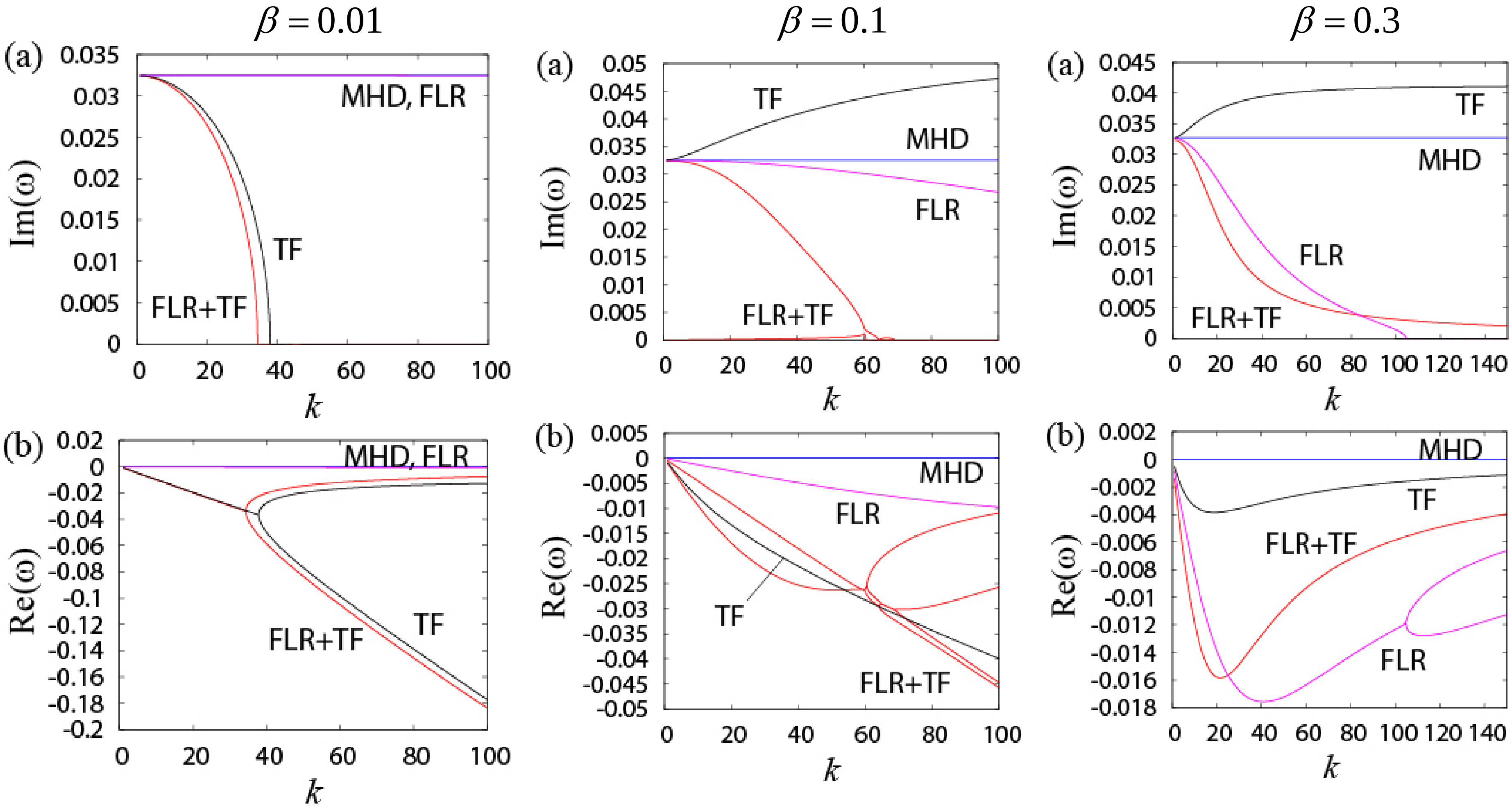




$$n_1=1.0,\quad x_1=-0.5,\quad x_2=0.5,\quad B_Z=1.0$$

$$p_1=1.0,\quad g=-0.01,\quad d_i/L=0.2,\quad \tau=p_i/p=0.5$$

Comparison of growth rates for different fluid models $(p_2 = 1.0)$



- Strong FLR stabilization occurs for high beta
- Two-fluid effect is stabilizing for low beta but destabilizing for high beta
- Coupling of FLR and two-fluid effects indicates strong stabilization for low beta but is less stabilizing for large wavenumber modes than the FLR effect
- For FLR+two-fluid case, RT is coupled with electron drift wave

Numerical analysis for two-fluid tearing mode in a slab $\left(\lambda_i = 0\right)$

$$\mathbf{B}_0 = \left[0, B_{0y}(x), B_{0z}(x)\right], \quad n_0 = const., \quad p_{e0} = const.$$

$$B_{y0} = \varepsilon_B B_0 \tanh\left(x / L\right), \quad B_{z0} = \sqrt{B_0^2 - B_{y0}^2}$$

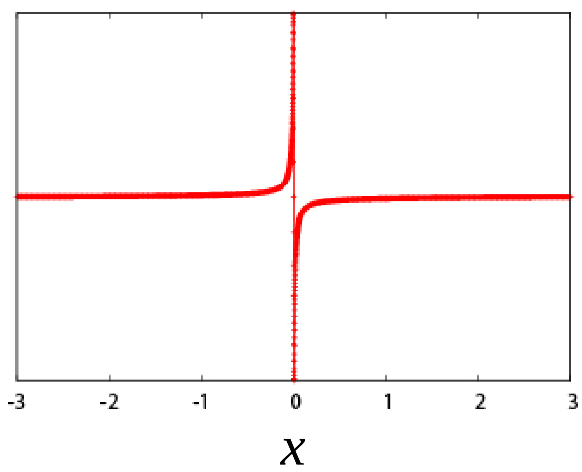
$$L = 0.75, \quad \varepsilon_B = 0.75$$

Boundary condition: $v_{1x}(\pm 3) = B_{1x}(\pm 3) = Q(\pm 3) = 0$

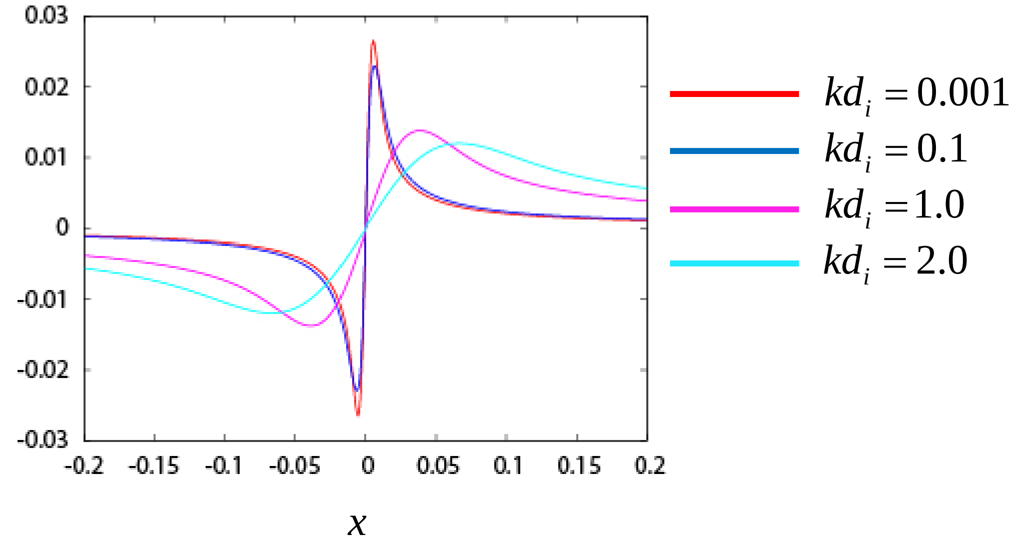
$$Q = B_{1z} + iB_{1x}B'_{0y} / \left(kB_{0z}\right)$$

$$\mathrm{Re}\left(v_{1x}\right) / \mathrm{Im}\left(B_{1x}(0)\right) \quad \left(S = 10^6, \beta = 0.05\right)$$

MHD



Two-fluid



Summary

RT mode

- Complete FLR stabilization disappears if beta value and pressuer gradient are small for equilibria with non-uniform magnetic field.
- Effects of FLR and two fluid on the growth rate and real frequency
  - Growth rate indicates complicated parameter dependence
- Growth rates for long wavelength modes for all cases and short wave for FLR case of eigenmode analysis agree with those of simulation results.

Tearing mode

- The eigenmode equations have been solved numerically for two-fluid tearing mode in a slab and a cylinder for benchmark with theory in a wide range of beta and ion skin depth.
- The effects of gyroviscosity with parallel heat flux based on the results for the parameter dependence of two-fluid tearing instability will be examined