Effect of parallel heat flux in gyroviscosity on tearing instability in two-fluid MHD

Atsushi Ito[1)] and J. J. Ramos[2)]

[1)]NIFS, [2)]MIT

伊藤淳[1)], J. J. Ramos[2)]

[1)]核融合研, [2)]MIT

Introduction

- Tearing mode instability in two-fluid MHD model
  - ➢ Drift tearing instability
    - Ion FLR effects on tearing mode instability
      [B. Coppi, Phys. Fluids **7**, 1501 (1964)]
    - Gyroviscosity is added to two-fluid MHD
    - Rotation of magnetic islands due to diamagnetic drifts in fusion plasmas was observed.
  - ➢ Contributions of heat flux cannot be neglected at low collisionality
    - Parallel heat fluxes are included in gyroviscosity
  - ➢ Formulation for linear stability analysis
    - We derive eigenmode equations for tearing instability in slab geometry including effects of parallel heat flux in the gyroviscous tensor.

Formulation for TMI in two-fluid model

- Fluid moment equations for collisionless magnetized plasmas
  [J.J. Ramos, Phys. Plasmas **12**, 052102, 112301 (2005), **15**, 082106 (2008)]

Equation of continuity:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = 0,$$

Equation of motion:

$$m_i n\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = \mathbf{j}\times\mathbf{B} - \sum_{s=i,e}\left[\nabla p_{s\perp} + \mathbf{B}\cdot\nabla\left(\frac{p_{s\|} - p_{s\perp}}{B^2}\right)\right] - \nabla\cdot\Pi_i^{gv},$$

Π_i^{gv}: ion gyroviscosity

Generalized Ohm's law:

$$\mathbf{E} + \mathbf{v}\times\mathbf{B} = \frac{1}{ne}\left\{\mathbf{j}\times\mathbf{B} - \left[\nabla p_{e\perp} + \mathbf{B}\cdot\nabla\left(\frac{p_{e\|} - p_{e\perp}}{B^2}\right)\right]\right\} + \eta\mathbf{j},$$

Faraday's law:

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla\times\mathbf{E} = 0,$$

Ampere's law:

$$\mu_0 \mathbf{j} = \nabla\times\mathbf{B},$$

Electron mass has been neglected.

- Gyroviscosity (Finite Larmor radius effect)

  Non-uniform magnetic field, anisotropic pressure and heat flux are taken into account.

$$\nabla \cdot \Pi_i^{gv} = \sum_{N=1}^{5} \left(\nabla \cdot \Pi_i^{gvN} \right)$$

- Pressure

  Equations for anisotropic pressure for ions:

$$\frac{1}{2}\frac{dp_{i\|}}{dt} + \frac{1}{2} p_{i\|} \nabla \cdot \mathbf{v} + p_{i\|} \mathbf{b} \cdot \left[\left(\mathbf{b} \cdot \nabla \right) \mathbf{v} \right] + \nabla \cdot \left(q_{iB\|} \mathbf{b} + \mathbf{q}_{iB\perp} \right) - 2\mathbf{q}_{iB\perp} \cdot \boldsymbol{\kappa}$$
$$+ q_{iT\|} \left(\mathbf{b} \cdot \nabla \right) \ln B + \mathbf{h}_{\perp} \cdot \left(\mathbf{b} \times \boldsymbol{\omega} \right) + q_{iT\|} \sigma = 0,$$

$$\frac{dp_{i\perp}}{dt} + 2 p_{i\perp} \nabla \cdot \mathbf{v} - p_{i\perp} \mathbf{b} \cdot \left[\left(\mathbf{b} \cdot \nabla \right) \mathbf{v} \right] + \nabla \cdot \left(q_{iT\|} \mathbf{b} + \mathbf{q}_{iT\perp} \right) - 2\mathbf{q}_{iB\perp} \cdot \boldsymbol{\kappa}$$
$$- q_{iT\|} \left(\mathbf{b} \cdot \nabla \right) \ln B + \mathbf{h}_{\perp} \cdot \left[2 \left(\mathbf{b} \cdot \nabla \right) \mathbf{v} \right] = 0,$$

- Perpendicular (diamagnetic) heat flux: $\boldsymbol{q}_{s\perp} \equiv \boldsymbol{q}_{sB\perp} + \boldsymbol{q}_{sT\perp} \quad (s = i, e)$

$$\mathbf{q}_{sB\perp} \equiv \frac{m_s}{2} \int \left(v_{\|} - \overline{v}_{\|} \right)^2 \left(\mathbf{v}_{\perp} - \overline{\mathbf{v}}_{\perp} \right) f d^3\mathbf{v} \simeq \frac{1}{e_s B} \mathbf{b} \times \left[\frac{1}{2} p_{s\perp} \nabla \left(\frac{p_{s\|}}{n} \right) + \frac{p_{s\|} \left(p_{s\|} - p_{s\perp} \right)}{n} \left(\mathbf{b} \cdot \nabla \mathbf{b} \right) \right],$$

$$\mathbf{q}_{sT\perp} \equiv \frac{m_s}{2} \int \left(v_{\perp} - \overline{v}_{\perp} \right)^2 \left(\mathbf{v}_{\perp} - \overline{\mathbf{v}}_{\perp} \right) f d^3\mathbf{v} \simeq \frac{1}{e_s B} \mathbf{b} \times \left[2 p_{s\perp} \nabla \left(\frac{p_{s\perp}}{n} \right) \right]$$

- Ion perpendicular heat fluxes

$$\mathbf{q}_{iB\perp} = \frac{1}{eB}\mathbf{b}\times\left[\frac{p_{i\perp}}{2}\nabla\left(\frac{p_{i\|}}{n}\right) + \frac{p_{i\|}\left(p_{i\|}-p_{i\perp}\right)}{n}\boldsymbol{\kappa} + 2m_i q_{iB\|}\left(\mathbf{b}\cdot\nabla\right)\mathbf{v} + m_i q_{iT\|}\mathbf{b}\times\boldsymbol{\omega}\right]$$

$$\mathbf{q}_{iT\perp} = \frac{2}{eB}\mathbf{b}\times\left[p_{i\perp}\nabla\left(\frac{p_{i\perp}}{n}\right) + 2m_i q_{iT\|}\left(\mathbf{b}\cdot\nabla\right)\mathbf{v}\right]$$

- Parallel heat flux: $\boldsymbol{q}_{s\|} \equiv \boldsymbol{q}_{sB\|} + \boldsymbol{q}_{sT\|} \quad \left(s = i, e\right)$

$$\mathbf{q}_{sB\|} \equiv \frac{m_s}{2}\int\left(v_\| - \bar{v}_{s\|}\right)^2\left(\mathbf{v}_\| - \bar{\mathbf{v}}_{s\|}\right) f d^3\mathbf{v}, \quad \mathbf{q}_{sT\|} \equiv \frac{m_i}{2}\int\left(v_\perp - \bar{v}_{s\perp}\right)^2\left(\mathbf{v}_\| - \bar{\mathbf{v}}_{s\|}\right) f d^3\mathbf{v}$$

- Equations for ion parallel heat fluxes

$$\frac{dq_{iB\|}}{dt}+q_{iB\|}\nabla\cdot\mathbf{v}+3q_{iB\|}\mathbf{b}\cdot\left[\left(\mathbf{b}\cdot\nabla\right)\mathbf{v}\right]+\frac{3p_{i\|}}{2m_i}\mathbf{b}\cdot\nabla\left(\frac{p_{i\|}}{n}\right)+\frac{1}{m_i}\mathbf{h}_\perp\cdot\left[\nabla\left(\frac{3p_{i\|}}{2n}\right)-\frac{3p_{i\|}}{n}\boldsymbol{\kappa}\right]$$

$$+3\mathbf{q}_{iB\perp}\cdot\left(\mathbf{b}\times\boldsymbol{\omega}\right)+\frac{s_B}{2m_i}=0,$$

$$\frac{dq_{iT\|}}{dt}+2q_{iT\|}\nabla\cdot\mathbf{v}+\frac{p_{i\|}}{m_i}\mathbf{b}\cdot\nabla\left(\frac{p_{i\perp}}{n}\right)-\frac{p_{i\perp}\left(p_{i\|}-p_{i\perp}\right)}{m_i n}\left(\mathbf{b}\cdot\nabla\right)\ln B+\frac{1}{m_i}\mathbf{h}_\perp\cdot\left[\nabla\left(\frac{p_{i\perp}}{2n}\right)+\frac{2p_{i\|}}{n}\boldsymbol{\kappa}\right]$$

$$+\mathbf{q}_{iT\perp}\cdot\left(\mathbf{b}\times\boldsymbol{\omega}\right)+4\mathbf{q}_{iB\perp}\cdot\left[\left(\mathbf{b}\cdot\nabla\right)\mathbf{v}\right]+\frac{p_{i\perp}^{\;2}}{m_i n}\sigma+\frac{s-s_B}{2m_i}+\frac{p_{i\perp}}{m_i}\nabla\cdot\left(\frac{\mathbf{h}_\perp}{n}\right)=0,$$

$$\mathbf{h}_\perp=\frac{m_i}{eB}\mathbf{b}\times\left[2p_{i\|}\left(\mathbf{b}\cdot\nabla\right)\mathbf{v}+p_{i\perp}\mathbf{b}\times\boldsymbol{\omega}+\nabla q_{iT\|}+2\left(q_{iB\|}-q_{iT\|}\right)\boldsymbol{\kappa}\right]$$

$$\boldsymbol{\omega}=\nabla\times\mathbf{v},\quad \boldsymbol{\kappa}=\left(\mathbf{b}\cdot\nabla\right)\mathbf{b}$$

- Assumptions for simplification
  - MHD ordering

$$v \sim v_{th}, \quad v_d \sim \delta v_{th}, \quad \delta \ll 1, \quad p_{e\parallel} = p_{e\perp}$$

    only first order diamagnetic terms are taken into account
  - Closure condition for fourth rank moments
    - Fourth rank moments are approximated with products of pressures.
  - Isotropic electron pressure
    - Equation for isotropic electron pressure is derived from electron heat flux equation.

$$\mathbf{B} \cdot \nabla \left(p_e / n \right) = 0$$

- Simplification of gyroviscous force

$$\nabla \cdot \Pi_i^{gv} = \sum_{N=1}^{5} \left(\nabla \cdot \Pi_i^{gvN} \right)$$

$$\nabla \cdot \Pi_i^{gv1} = -m_i n \mathbf{v}_{*i} \cdot \nabla \mathbf{v} - \nabla \chi_v - \nabla \times \left\{ \frac{m_i p_{i\perp}}{eB} \left\{ \left(\mathbf{b} \cdot \nabla \right) \mathbf{v} + \frac{1}{2} \left\{ \nabla \cdot \mathbf{v} - 3\mathbf{b} \cdot \left[\left(\mathbf{b} \cdot \nabla \right) \mathbf{v} \right] \right\} \mathbf{b} \right\} \right\}$$

$$+ \left(\mathbf{B} \cdot \nabla \right) \left\{ \frac{m_i p_{i\perp}}{eB^2} \mathbf{b} \times \left[3 \left(\mathbf{b} \cdot \nabla \right) \mathbf{v} + \mathbf{b} \times \boldsymbol{\omega} \right] + \frac{\chi_v}{B} \mathbf{b} \right\},$$

$$\mathbf{v}_{*i} \equiv -\frac{1}{en} \nabla \times \left(\frac{p_i}{B^2} \mathbf{B} \right), \qquad \chi_v \equiv \frac{m_i p_i}{2eB} \mathbf{b} \cdot \boldsymbol{\omega},$$

$$\nabla\cdot\Pi_i^{gv2} = \frac{m_i}{e}\left[\nabla\times\left(\frac{\mathbf{b}}{B}\right)\right]\cdot\nabla\left[\left(q_{T\parallel}-\frac{\alpha j_\parallel}{B}\right)\mathbf{b}+\frac{\boldsymbol{q}_{T\perp}}{2}\right]-\nabla\chi_q-\nabla\times\left\{\frac{m_i}{eB}\left\{\left(\mathbf{b}\cdot\nabla\right)\left[\left(q_{T\parallel}-\frac{\alpha j_\parallel}{B}\right)\mathbf{b}+\frac{\boldsymbol{q}_{T\perp}}{2}\right]\right.\right.$$

$$\left.\left.+\frac{1}{2}\left\{\nabla\cdot\left[\left(q_{T\parallel}-\frac{\alpha j_\parallel}{B}\right)\mathbf{b}+\frac{\boldsymbol{q}_{T\perp}}{2}\right]-3\mathbf{b}\cdot\left\{\left(\mathbf{b}\cdot\nabla\right)\left[\left(q_{T\parallel}-\frac{\alpha j_\parallel}{B}\right)\mathbf{b}+\frac{\boldsymbol{q}_{T\perp}}{2}\right]\right\}\right\}\mathbf{b}\right\}\right\}$$

$$+\left(\mathbf{B}\cdot\nabla\right)\left\{\frac{m_i}{eB^2}\mathbf{b}\times\left\{3\left(\mathbf{b}\cdot\nabla\right)\left[\left(q_{T\parallel}-\frac{\alpha j_\parallel}{B}\right)\mathbf{b}+\frac{\boldsymbol{q}_{T\perp}}{2}\right]+\mathbf{b}\times\left\{\nabla\times\left[\left(q_{T\parallel}-\frac{\alpha j_\parallel}{B}\right)\mathbf{b}+\frac{\boldsymbol{q}_{T\perp}}{2}\right]\right\}\right\}+\frac{\chi_q}{B}\mathbf{b}\right\},$$

$$\chi_q = \frac{m_i}{2eB}\mathbf{b}\cdot\left\{\nabla\times\left[\left(q_{T\parallel}-\alpha j_\parallel / B\right)\mathbf{b}+\frac{\boldsymbol{q}_{T\perp}}{2}\right]\right\}, \quad \alpha = \frac{p_{i\perp}\left(p_{i\parallel}-p_{i\perp}\right)}{2neB}$$

MHD ordering

$$\nabla\cdot\Pi_i^{gv2} \simeq \frac{m_i}{e}\left[\nabla\times\left(\frac{\mathbf{b}}{B}\right)\right]\cdot\nabla\left(q_{iT\parallel}\mathbf{b}\right)-\nabla\chi_q$$

$$-\nabla\times\left\{\frac{m_i}{eB}\left\{\left(\mathbf{b}\cdot\nabla\right)\left(q_{iT\parallel}\mathbf{b}\right)+\frac{1}{2}\left\{\nabla\cdot\left(q_{iT\parallel}\mathbf{b}\right)-3\mathbf{b}\cdot\left[\left(\mathbf{b}\cdot\nabla\right)\left(q_{iT\parallel}\mathbf{b}\right)\right]\right\}\mathbf{b}\right\}\right\}$$

$$+\left(\mathbf{B}\cdot\nabla\right)\left\{\frac{m_i}{eB^2}\mathbf{b}\times\left[3\left(\mathbf{b}\cdot\nabla\right)\left(q_{iT\parallel}\mathbf{b}\right)+\mathbf{b}\times\left(\nabla\times\left(q_{iT\parallel}\mathbf{b}\right)\right)\right]+\frac{\chi_q}{B}\mathbf{b}\right\},$$

$$\chi_q \simeq \frac{m_i}{2eB}\mathbf{b}\cdot\left[\nabla\times\left(q_{T\parallel}\mathbf{b}\right)\right],$$

Parallel heat fluxes are included in $\nabla\cdot\Pi_i^{gv2}$

$$\nabla\cdot\Pi_i^{gv3} = \nabla\times\left\{\mathbf{B}\times\left[\frac{m_i}{eB^2}\left(\mathbf{c}+\mathbf{d}\right)\right]\right\} + \left(\mathbf{B}\cdot\nabla\right)\left[\frac{2m_i}{eB^2}\left(\mathbf{c}+\mathbf{d}\right)\right],$$

$$\mathbf{c} = \left(2q_{iB\parallel} - 3q_{iT\parallel}\right)\boldsymbol{\kappa} + \left(\frac{p_{i\parallel}-p_{i\perp}}{B}\right)\left\{2\left(\mathbf{B}\cdot\nabla\right)\mathbf{v} - \nabla\times\left[\frac{1}{en}\nabla p_{i\perp} + \frac{1}{en}\left(\mathbf{B}\cdot\nabla\right)\left(\frac{p_{i\parallel}-p_{i\perp}}{B}\mathbf{b}\right)\right]\right\},$$

$$\mathbf{d} = \frac{3\alpha j_\parallel}{B}\boldsymbol{\kappa} + \nabla\times\left[\left(2\mathbf{q}_{B\perp} - \frac{1}{2}\mathbf{q}_{T\perp}\right)\times\mathbf{b} - \alpha\left(\nabla\cdot\mathbf{b}\right)\mathbf{b}\right] + 2\left\{\left[2\mathbf{q}_{B\perp} - \frac{1}{2}\mathbf{q}_{T\perp} + \nabla\times\left(\alpha\mathbf{b}\right)\right]\cdot\nabla\right\}\mathbf{b}$$

MHD ordering

$$\nabla\cdot\Pi_i^{gv3} \simeq \nabla\times\left\{\mathbf{B}\times\left\{\frac{m_i}{eB^2}\left[\left(2\boldsymbol{q}_{iB\parallel} - 3\boldsymbol{q}_{iT\parallel}\right)\boldsymbol{\kappa} + 2\left(\frac{p_{i\parallel}-p_{i\perp}}{B}\right)\left(\mathbf{B}\cdot\nabla\right)\mathbf{v}\right]\right\}\right\}$$
$$+\left(\mathbf{B}\cdot\nabla\right)\left\{\frac{2m_i}{eB^2}\left[\left(2\boldsymbol{q}_{iB\parallel} - 3\boldsymbol{q}_{iT\parallel}\right)\boldsymbol{\kappa} + 2\left(\frac{p_{i\parallel}-p_{i\perp}}{B}\right)\left(\mathbf{B}\cdot\nabla\right)\mathbf{v}\right]\right\},$$
$$\nabla\cdot\Pi_i^{gv4} \simeq \nabla\cdot\Pi_i^{gv5} \simeq 0,$$

Linearization

$$\left(\nabla\cdot\Pi_i^{gv3}\right)_1 = 0$$

Linear eigenmode equations

- Equilibrium
  - Static, slab equilibrium

$$\mathbf{v}_0 = 0, \quad \mathbf{B}_0 = \left[0, B_{0y}(x), B_{0z}(x)\right], \quad n_0 = n_0(x)$$

  - Constant electron temperature

$$T_e = const.$$

  - Isotropic equilibrium ion pressure

$$p_{i||0} = p_{i\perp 0} = p_{i0}(x), \quad q_{iB||0} = q_{iT||0} = 0$$

  - Finite-beta equilibrium

$$\frac{d}{dx}\left[\frac{1}{2\mu_0}\left(B_{0y}^2 + B_{0z}^2\right) + p_{i0} + p_{e0}\right] = 0$$

- Linearization

$$f_1 = f_1(x)\exp\left[-i(\omega t - ky)\right]$$

- Normalization

$$\omega = \left(V_A / L\right)\bar{\omega}, \quad k = \bar{k} / L, \quad v = V_A \bar{v}, \quad n = n_* \bar{n}, \quad B = B_* \bar{B},$$

$$p = \left(B_*^2 / \mu_0\right)\bar{p}, \quad \Pi_i^{gv} = \left(B_*^2 / \mu_0\right)\bar{\Pi}_i^{gv}$$

$$V_A = \frac{B_*}{\sqrt{\mu_0 n_* m_i}}, \quad d_i = \sqrt{\frac{m_i}{\mu_0 n_* e^2}}$$

- Eigenmode equations

$$-in_0\omega v_{1x} = -B_{0z}B'_{1z} - B'_{0z}B_{1z} - \frac{i}{k}\left(B_{0y}B''_{1x} + B'_{0y}B'_{1x}\right) + ikB_{0y}B_{1x} - p'_{i\perp 1} - p'_{e1} - \lambda_i\left(\nabla\cdot\Pi_i^{gv}\right)_{1x}$$

$$-in_0\omega v_{1y} = -ikB_{0z}B_{1z} + B'_{0y}B_{1x} - \frac{ik}{B_0^2}\left(B_{0y}^2 p_{i\|1} + B_{0z}^2 p_{i\perp 1}\right) - ikp_{e1} - \lambda_i\left(\nabla\cdot\Pi_i^{gv}\right)_{1y}$$

$$-in_0\omega v_{1z} = ikB_{0y}B_{1z} + B'_{0z}B_{1x} - \frac{ikB_{0y}B_{0z}}{B_0^2}\left(p_{i\|1} - p_{i\perp 1}\right) - \lambda_i\left(\nabla\cdot\Pi_i^{gv}\right)_{1z}$$

$$-i\left(\omega - \lambda_H\frac{d_i}{L}\frac{kB'_{0z}}{n_0} + i\frac{\eta k^2}{\mu_0 L V_A}\right)B_{1x} - i\frac{\eta}{\mu_0 L V_A}B''_{1x} - \lambda_H\frac{d_i}{L}\frac{k^2 B_{0y}}{n_0}B_{1z} - ikB_{0y}v_{1x} = 0$$

$$-i\left(\omega - \lambda_H\frac{d_i}{L}\frac{kB_{0z}n'_0}{n_0^2} + i\frac{\eta k^2}{\mu_0 L V_A}\right)B_{1z} + B_{0z}v'_{1x} + B'_{0z}v_{1x} - ik\left(-B_{0z}v_{1y} + B_{0y}v_{1z}\right) - i\frac{\eta}{\mu_0 L V_A}B''_{1z}$$

$$-\lambda_H\frac{d_i}{L}\frac{1}{n_0}\left\{\left(\frac{n'_0}{n_0}B'_{0y} - B''_{0y} - k^2 B_{0y}\right)B_{1x} + B_{0y}B''_{1x} + \frac{kB_0 B'_0}{n_0\omega}\left[n'_0 v_{1x} + n_0\left(v'_{1x} + ikv_{1y}\right)\right]\right\} = 0$$

$$
\begin{aligned}
(\lambda_H, \lambda_i) = (0,0) &\quad \Rightarrow \quad \text{Single - fluid MHD} \\
= (1,0) &\quad \Rightarrow \quad \text{Hall MHD} \\
= (1,1) &\quad \Rightarrow \quad \text{FLR two - fluid MHD}
\end{aligned}
$$

$$
\begin{aligned}
\lambda_{i||} = 0 &\quad \Rightarrow \quad \text{adiabatic ion pressure} \\
= 1 &\quad \Rightarrow \quad \text{ion pressure with parallel heat flux}
\end{aligned}
$$

$$
-i\omega n_1 + n_0' v_{1x} + n_0 \left(v_{1x}' + ikv_{1y} \right) = 0
$$

$$
p_{e1} = \frac{p_{e0}}{n_0} n_1
$$

$$
\begin{aligned}
&\frac{1}{2}\left(-i\omega p_{i||1} + p_{i0}' v_{x1}\right) + \frac{p_{i0}}{2}\left(v_{x1}' + ikv_{y1}\right) + i\frac{kB_{0y}' p_{i0}}{B_0^2}\left(B_{0y} v_{y1} + B_{0z} v_{z1}\right) + i\frac{\lambda_{||} kB_{0y}}{B_0} q_{iB||1} \\
&+ i\lambda_i \frac{kd_i}{2L}\left\{\left[-\frac{B_{0z} p_{i0}}{B_0^2 n_0}\left(p_{i||1} - \frac{p_{i0}}{n_0} n_1\right)\right]' + \frac{B_{0z}}{B_0^2}\left(\frac{p_{i0}}{n_0}\right)'\left(p_{i\perp 1} - 2p_{i0}\frac{B_{0y} B_{y1} + B_{0z} B_{z1}}{B_0^2}\right)\right. \\
&\left.+ \frac{p_{i0}}{B_0^2}\left\{\left(\frac{p_{i0}}{n_0}\right)' B_{z1} + B_{0z}\left[\frac{p_{i||1}'}{n_0} - \frac{n_0'}{n_0^2} p_{i||1} - \frac{p_{i0}}{n_0^2} n_1' - \left(\frac{p_{i0}}{n_0}\right)' n_1\right]\right\}\right\} \\
&- \frac{\lambda_i d_i}{L}\frac{p_{i0}}{B_0^2}\left(\frac{p_{i0}}{n_0}\right)'\left\{\left(B_{0z} B_{0y}' + B_{0y} B_{0z}'\right)\frac{B_{x1}}{B_0^2} - \frac{B_{0y} B_{0z}}{B_0^2} B_{x1}' - i\frac{kB_{0y}^2}{B_0^2} B_{z1}\right\} = 0
\end{aligned}
$$

$$-i\omega p_{i\perp 1}+p_{i0}'v_{x1}+2p_{i0}\left(v_{x1}'+ikv_{y1}\right)-i\frac{kB_{0y}'p_{i0}}{B_0^2}\left(B_{0y}v_{y1}+B_{0z}v_{z1}\right)+i\frac{\lambda_{\|}kB_{0y}}{B_0}q_{iT\|1}$$

$$+i\lambda_i\frac{2kd_i}{L}\left\{\left[-\frac{B_{0z}p_{i0}}{B_0^2n_0}\left(p_{i\perp 1}-\frac{p_{i0}}{n_0}n_1\right)\right]'+\frac{B_{0z}}{B_0^2}\left(\frac{p_{i0}}{n_0}\right)'\left(p_{i\perp 1}-2p_{i0}\frac{B_{0y}B_{y1}+B_{0z}B_{z1}}{B_0^2}\right)\right.$$

$$\left.+\frac{p_{i0}}{B_0^2}\left\{\left(\frac{p_{i0}}{n_0}\right)'B_{z1}+B_{0z}\left[\frac{p_{i\perp 1}'}{n_0}-\frac{n_0'}{n_0^2}p_{i\perp 1}-\frac{p_{i0}}{n_0^2}n_1'-\left(\frac{p_{i0}}{n_0}\right)'n_1\right]\right\}\right\}$$

$$-\frac{\lambda_i d_i}{L}\frac{p_{i0}}{B_0^2}\left(\frac{p_{i0}}{n_0}\right)'\left\{\left(B_{0z}B_{0y}'+B_{0y}B_{0z}'\right)\frac{B_{x1}}{B_0^2}-\frac{B_{0y}B_{0z}}{B_0^2}B_{x1}'-i\frac{kB_{0y}^2}{B_0^2}B_{z1}\right\}=0$$

$$-i\omega q_{iB\|1}+\frac{3ikp_{i0}}{2n_0}\left(p_{i\|1}-\frac{p_{i0}}{n_0}n_1\right)-i\lambda_i\frac{d_i}{L}\frac{kB_{0z}p_{i0}'}{n_0B_0^2}q_{iB\|1}-i\lambda_i\frac{d_i}{L}\frac{3kp_{i0}}{2n_0B_0^2}\left(\frac{2B_0'B_{0z}}{B_0}-B_{0z}'\right)q_{iT\|1}$$

$$-i\lambda_i\frac{d_i}{L}\frac{3kp_{i0}B_{0y}}{B_0^3}\left(\frac{p_{i0}}{n_0}\right)'\left(B_{0z}v_{1y}-B_{0y}v_{1z}\right)=0$$

$$
\begin{aligned}
&-i\omega q_{iT\|1}+\frac{3ikp_{i0}}{2n_0}\left(p_{i\|1}-\frac{p_{i0}}{n_0}n_1\right)-i\lambda_i\frac{d_i}{L}\frac{kB_{0z}p_{i0}'}{n_0B_0^2}q_{iT\|1}+i\lambda_i\frac{d_i}{L}\frac{kp_{i0}}{n_0B_0^2}\left(\frac{2B_0'B_{0z}}{B_0}-B_{0z}'\right)q_{iT\|1}\\
&+2\lambda_i\frac{d_i}{L}p_{i0}\left\{i\frac{kp_{i0}B_{0y}}{n_0B_0^3}\left(B_{0y}v_{1z}'-B_{0z}v_{1y}'\right)+i\left(\frac{kp_{i0}B_{0y}^2}{n_0B_0^3}\right)'v_{1z}-i\left(\frac{kp_{i0}B_{0y}B_{0z}}{n_0B_0^3}\right)'v_{1y}-\frac{k^2p_{i0}B_{0y}B_{0z}}{n_0B_0^3}v_{1x}\right\}\\
&+\lambda_i\frac{d_i}{L}\frac{p_{i0}^2}{n_0}\left\{\frac{ikB_0'}{B_0^2}v_{1z}+\frac{ikB_{0y}}{B_0^3}\left[-B_{0y}v_{1z}'+B_{0z}\left(v_{1y}'-ikv_{1x}\right)\right]\right\}\\
&+\lambda_i\frac{d_i}{L}\frac{p_{i0}^2}{4n_0B_0^2}\left\{B_{0y}\left[2\left(\frac{B_{0z}}{B_0}\right)'v_{1x}'-ik\left(\frac{B_{0y}}{B_0}\right)'\frac{B_{0z}^2}{B_0^2}v_{1z}+ik\left(\frac{B_{0z}}{B_0}\right)'\frac{B_{0y}B_{0z}}{B_0^2}v_{1z}\right]\right.\\
&\left.+B_{0z}\left[\left(\frac{B_{0y}}{B_0}\right)'\left[2ik\frac{B_{0z}^2}{B_0^2}v_{1y}-2v_{1x}'-ik\frac{B_{0y}B_{0z}}{B_0^2}v_{1z}\right]+ik\left(\frac{B_{0z}}{B_0}\right)'\frac{B_{0y}}{B_0^2}\left(B_{0y}v_{1z}-2B_{0z}v_{1y}\right)\right]\right\}=0
\end{aligned}
$$

$$\left(\nabla\cdot\Pi_i^{gv1}\right)_{1x}=\frac{d_i}{L}\left\{-ik\left(\frac{B_{0z}p_{i0}}{B_0^2}\right)'v_{1x}+\frac{k^2B_{0y}p_{i0}}{B_0^2}v_{1z}-i\frac{kB_{0z}p_{i0}}{2B_0^2}\left[v'_{x1}+ikv_{1y}-\frac{3ikB_{0y}}{B_0^2}\left(B_{0y}v_{y1}+B_{0z}v_{z1}\right)\right]\right.$$
$$\left.-\left\{\frac{p_{i0}}{2B_0^2}\left[-B_{0y}v'_{z1}+B_{0z}\left(v'_{y1}-ikv_{x1}\right)\right]\right\}'-\frac{3k^2B_{0y}^2p_{i0}}{B_0^4}\left(B_{0y}v_{z1}-B_{0z}v_{y1}\right)+\frac{k^2B_{0y}p_{i0}}{B_0^2}v_{z1}\right\}$$

$$\left(\nabla\cdot\Pi_i^{gv1}\right)_{1y}=\frac{d_i}{L}\left\{ik\left[-\left(\frac{B_{0z}p_{i0}}{B_0^2}\right)'v_{1y}+\left(\frac{B_{0y}p_{i0}}{B_0^2}\right)'v_{1z}+\frac{B_{0y}p_{i0}}{B_0^2}v'_{z1}\right]\right.$$
$$+\frac{kB'_{0z}p_{i0}}{2B_0^2}\left[v'_{x1}+ikv_{1y}-\frac{3ikB_{0y}}{B_0^2}\left(B_{0y}v_{y1}+B_{0z}v_{z1}\right)\right]$$
$$+\frac{B_{0z}}{2}\left\{\frac{p_{i0}}{B_0^2}\left[v'_{x1}+ikv_{y1}-\frac{3ikB_{0y}}{B_0^2}\left(B_{0y}v_{y1}+B_{0z}v_{z1}\right)\right]\right\}'$$
$$-i\frac{kB_{0z}^2p_{i0}}{2B_0^4}\left[-B_{0y}v'_{z1}+B_{0z}\left(v'_{y1}-ikv_{x1}\right)\right]$$
$$\left.-\frac{3k^2B_{0y}^2B_{0z}p_{i0}}{B_0^4}v_{x1}+i\frac{kB_{0y}B_{0z}p_{i0}}{B_0^4}\left[B_{0z}v'_{z1}+B_{0y}\left(v'_{y1}-ikv_{x1}\right)\right]\right\}$$

$$\left(\nabla\cdot\Pi_i^{gv1}\right)_{1z} = \frac{d_i}{L}\Bigg\{-ik\left[\left(\frac{B_{0z}p_{i0}}{B_0^2}\right)' v_{1z} + \left(\frac{B_{0y}p_{i0}}{B_0^2}\right)' v_{1y} + \frac{B_{0y}p_{i0}}{B_0^2}\left(v'_{y1} - ikv_{1x}\right)\right]$$

$$-\frac{B'_{0y}p_{i0}}{2B_0^2}\left[v'_{x1} + ikv_{1y} - \frac{3ikB_{0y}}{B_0^2}\left(B_{0y}v_{y1} + B_{0z}v_{z1}\right)\right]$$

$$-\frac{B_{0y}}{2}\left\{\frac{p_{i0}}{B_0^2}\left[v'_{x1} + ikv_{y1} - \frac{3ikB_{0y}}{B_0^2}\left(B_{0y}v_{y1} + B_{0z}v_{z1}\right)\right]\right\}'$$

$$+i\frac{kB_{0y}B_{0z}p_{i0}}{2B_0^4}\left[-B_{0y}v'_{z1} + B_{0z}\left(v'_{y1} - ikv_{x1}\right)\right]$$

$$+\frac{3k^2B_{0y}^3p_{i0}}{B_0^4}v_{x1} - i\frac{kB_{0y}^2p_{i0}}{B_0^4}\left[B_{0z}v'_{z1} + B_{0y}\left(v'_{y1} - ikv_{x1}\right)\right]\Bigg\}$$

$$\left(\nabla\cdot\Pi_i^{gv2}\right)_{1x} = \frac{d_i}{L}\left\{\frac{B_{0y}B'_{0z} - B'_{0y}B_{0z}}{2B_0^3}q'_{iT\parallel1} - \left[\left(\frac{-B_{0y}B'_{0z} + B'_{0y}B_{0z}}{2B_0^3}\right)' - \frac{k^2B_{0y}B_{0z}}{B_0^3}\right]q_{iT\parallel1}\right\}$$

$$\left(\nabla\cdot\Pi_i^{gv2}\right)_{1x} = \frac{ik}{B_0^3}\frac{d_i}{L}\left\{B_{0y}B_{0z}q'_{iT\parallel1} + \frac{1}{B_0}\left[B_{0y}\left(2B'_0B_{0z} - B_0B'_{0z}\right) + \frac{B_{0z}^2}{2B_0}\left(B_{0y}B'_{0z} - B_{0z}B'_{0y}\right)\right]q_{iT\parallel1}\right\}$$

$$\left(\nabla\cdot\Pi_i^{gv2}\right)_{1z} = i\frac{d_i}{L}\frac{k}{B_0^3}\left\{-B_{0y}^2q'_{iT\parallel1} + \frac{1}{B_0}\left[B_{0z}\left(2B'_0B_{0z} - B_0B'_{0z}\right) + \frac{B_{0y}B'_{0z}}{2B_0}\left(-B_{0y}B'_{0z} + B'_{0y}B_{0z}\right)\right]q_{iT\parallel1}\right\}$$

- Finite beta equilibrium

$$p_{i0}(x) = \tau\beta\left\{1+\varepsilon_p\left[1+\tanh\left(x/L_p\right)\right]\right\}$$

$$p_{e0}(x) = (1-\tau)\beta\left\{1+\varepsilon_n\left[1+\tanh\left(x/L_n\right)\right]\right\}$$

$$n_0(x) = n_0\left\{1+\varepsilon_n\left[1+\tanh\left(x/L_n\right)\right]\right\}$$

$$B_{0y}(x) = \varepsilon_B\sqrt{1+\beta}B_0\tanh\left(x/L\right)$$

$$B_{0z}(x) = \sqrt{(1+\beta)B_0^2 - B_{0y}^2(x) - \left[p_{i0}(x)+p_{e0}(x)\right]}$$

- MHD, incompressible

$$n_0\omega\left(v''_{1x} - k^2 v_{1x}\right) + k\left[B_{0y}B''_{1x} - \left(B''_{0y} + k^2 B_{0y}\right)B_{1x}\right] = 0$$

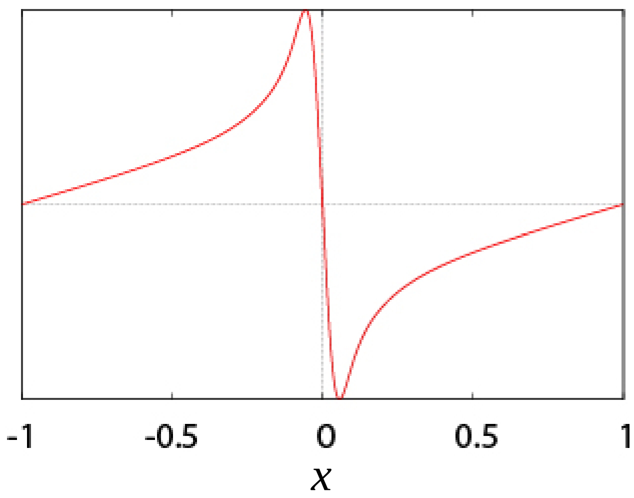
$$\left(\omega + i\frac{\eta k^2}{\mu_0 L V_A}\right)B_{1x} - i\frac{\eta}{\mu_0 L V_A}B''_{1x} + kB_{0y}v_{1x} = 0$$

- Boundary condition: $v_{1x}\left(\pm 1\right) = B_{1x}\left(\pm 1\right) = 0$
- Numerical solution

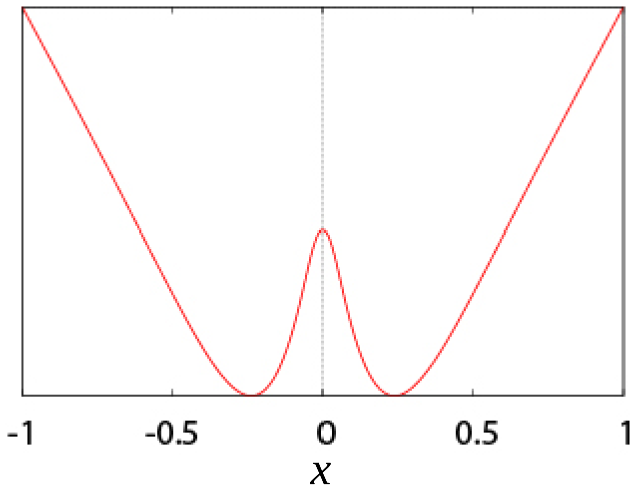
$$\frac{\eta}{\mu_0 L V_A} = 1.0\times 10^{-4}, \quad \varepsilon_B = 1/3, \quad \varepsilon_p = \varepsilon_n = 0, \quad \beta = 0, \quad L = 0.25, \quad B_0 = 1$$

$$\omega = 0.018\mathrm{i}$$

$\mathrm{Re}\left(v_{1x}\right)$



$\mathrm{Im}\left(B_{1x}\right)$



Summary

- Formulation for linear stability analysis
  We have derived eigenmode equations for tearing instability in slab geometry including effects of parallel heat flux in the gyroviscous tensor.

- The eigenmode equation will be solved numerically.