MHD Instability Excited by Interplay between Resistive Wall Mode and Stable MHD Modes in Rotating Tokamak Plasmas

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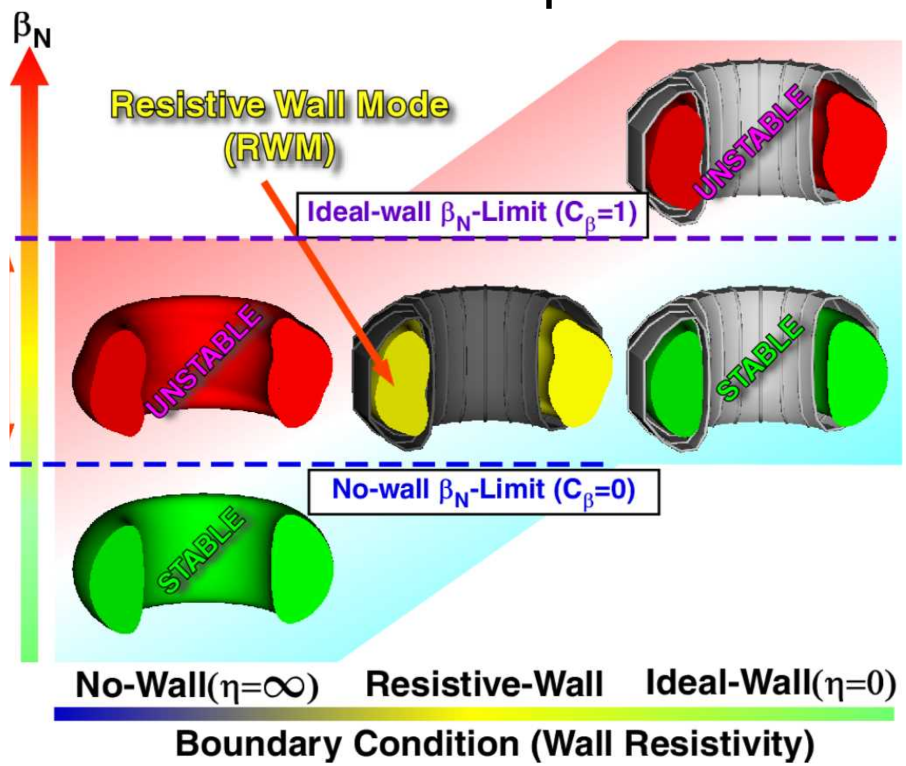
MHD modes in high-β tokamaks

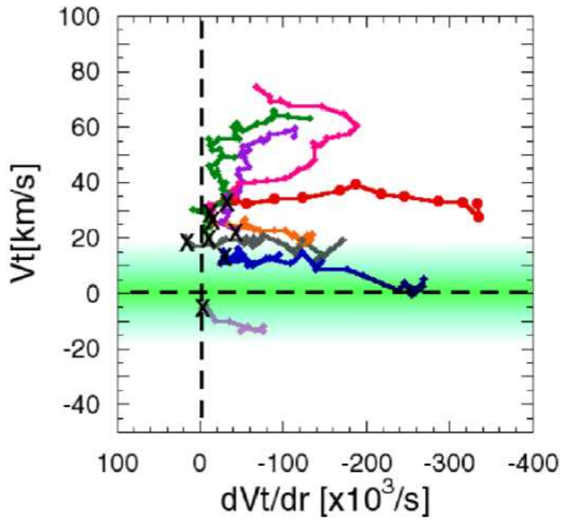
- For realizing economical fusion reactor, it is important to develop a MHD equilibrium with high-β; β is the ratio between plasma thermal pressure and magnetic pressure.
  In such a high-β equilibrium, MHD modes sometimes become unstable, and a long wavelength mode induces “disruption”.
- Such a MHD mode is usually stabilized by surrounding the plasma with conducting wall.
- However, if the conducting wall has resistivity, so-called resistive wall mode (RWM) becomes unstable[Strait PoP1994 etc.].
  => Disruption



Rotation is responsible for RWM stability

- About 20 years ago, theoretical papers identified that RWM can be stabilized by plasma toroidal rotation [Bondeson PoP1994].
- Rotation stabilization has been observed experimentally in many tokamaks [LaHaye PoP2004 etc.].
- However, high-β plasma discharges are sometimes terminated by MHD instability even when plasma rotation successfully stabilize RWM[Matsunaga IAEA2008, Sabbagh NF2010].
- For realizing high-β steady-state fusion reactor, it is necessary to understand the reason why such disruptive MHD instability appears.

Does plasma rotation always stabilize RWM?



RWM experimental results in JT-60U [Matsunaga IAEA2008]

Theoretical works predicted rotation can destabilize RWM

- In a cylindrical plasma, several theoretical works identified that RWM can be destabilized due to
  a. coupling between RWM and stable MHD discrete mode [Finn PoP1996, Lashmore-Davies PoP2001/JPP2005].
  b. wall resistivity destabilizing negative energy modes[Lashmore-Davies JPP2005/Hirota PST2009].
  c. resonance between stable MHD discrete mode and continuum when their energies have opposite signs [Hirota PST2009].

Do these mechanisms excite RWM in tokamaks?

Basic equations

The ideal MHD stability code, MINERVA[Aiba CPC2009] solves the Frieman-Rotenberg equation [Frieman RMP1960] with sound wave damping force modelling ion Landau damping[Chu PoP1995]

$$\rho \frac{\partial^2 \xi}{\partial t^2} + 2\rho(\mathbf{u}\cdot\nabla)\frac{\partial \xi}{\partial t} = \mathbf{F}(\xi) + \mathbf{F}_{S.D.}(v_{//}),$$

$$\mathbf{F}(\xi) = \mathbf{F}_s(\xi) + \nabla \otimes [\rho\xi \otimes (\mathbf{u}\cdot\nabla)\mathbf{u} - \rho\mathbf{u} \otimes (\mathbf{u}\cdot\nabla)\xi]$$

$\mathbf{F}_s$: Force operator (same vector form as that in static equilibrium case)

$\mathbf{u}$: Equilibrium rotation velocity

$\mathbf{F}_{\mathrm{S.D.}}(v_{//}) = -\kappa_{//} |k_{//} v_{th}| \rho v_{//} \mathbf{B}/B$: Sound wave damping force

$$v_{//} = \frac{\partial \xi_{//}}{\partial t} + ((\mathbf{u}\cdot\nabla)\xi)\cdot\mathbf{B}/B$$

To identify RWM stability in tokamak plasmas, RWMaC [Shiraishi NF2014] is implemented to MINERVA[Aiba PoP2011].

$$\underbrace{\left\langle \xi \middle| \rho \frac{\partial^2 \xi}{\partial t^2} \right\rangle + 2\langle \xi | \rho(\boldsymbol{u}\cdot\nabla)\frac{\partial \xi}{\partial t} - \langle \xi | \boldsymbol{F}(\xi) + \boldsymbol{F}_{S.D.}(v_{||}) \rangle}_{\text{MINERVA}} + \underbrace{\delta W_V - \frac{\tau_{A0}}{\tau_w}\frac{\partial D_w}{\partial t}}_{\text{RWMaC}} = 0$$

δW_V: vacuum energy

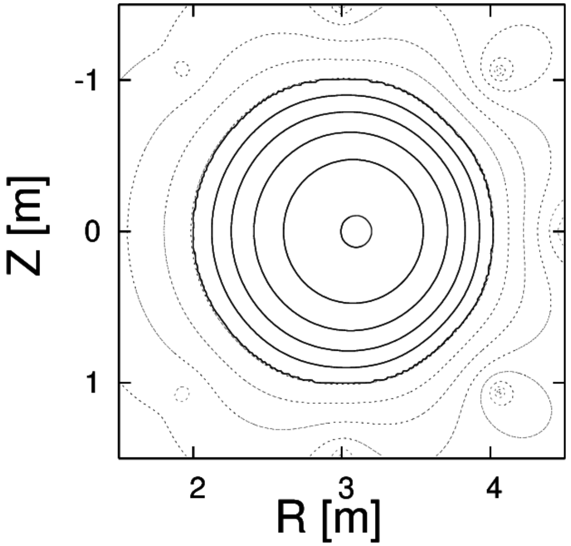
D_w : energy dissipated in the resistive wall

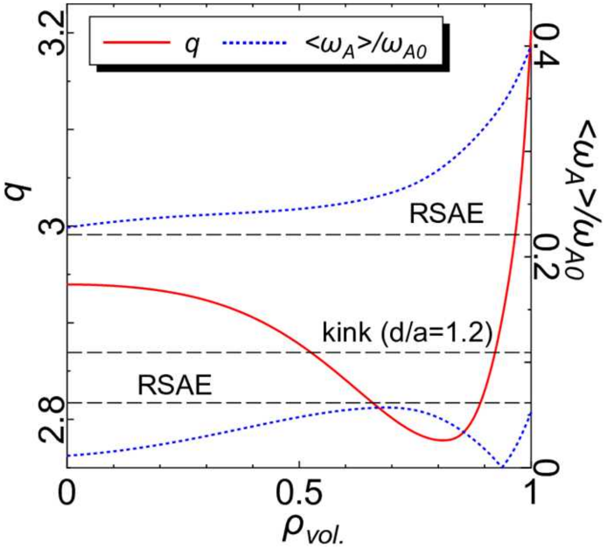
Analysis for the simplest case

To examine whether RWM in a torus plasma is destabilized by plasma rotation as in a cylindrical plasma, we try to simplify the problem/equilibrium.

Plasma parameters

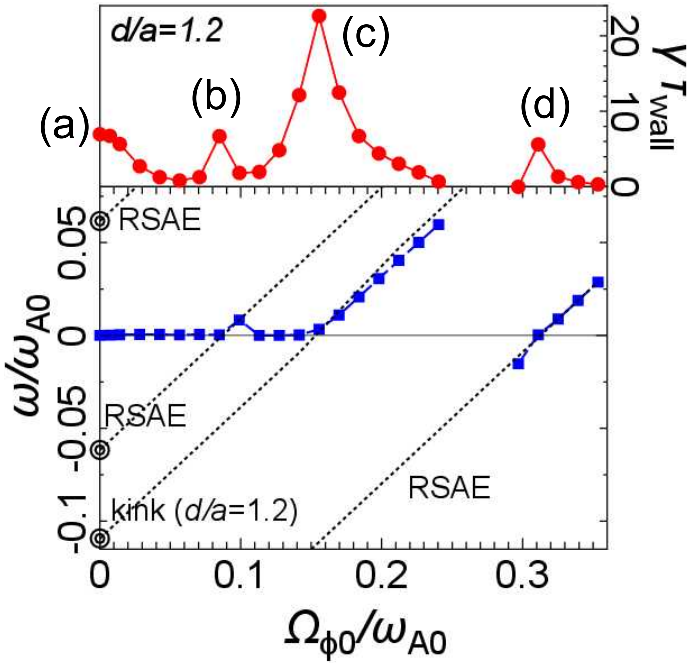
$$\beta_p = 0.3$$
$$B_T = 1.0[T]$$
$$I_p = 0.6[MA]$$
$$\frac{d}{a} = 1.2$$





- RWM stability is analyzed in this plasma with a variety of rigid toroidal rotation.
- At first, plasma compression is neglected.

Unstable mode appears due to coupling between RWM and stable modes



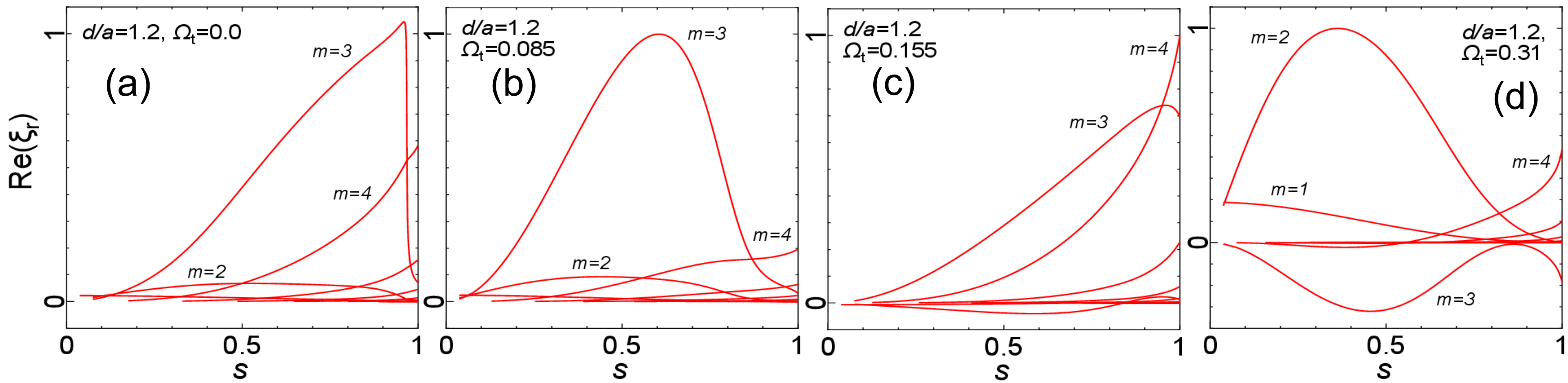
In this simplest case, RWM destabilization by rotation is observed.

There are 3 resonant modes.

b. RSAE ($m = 3$)
c. External kink (mainly $m = 4$)
d. GAE ($m = 2$)

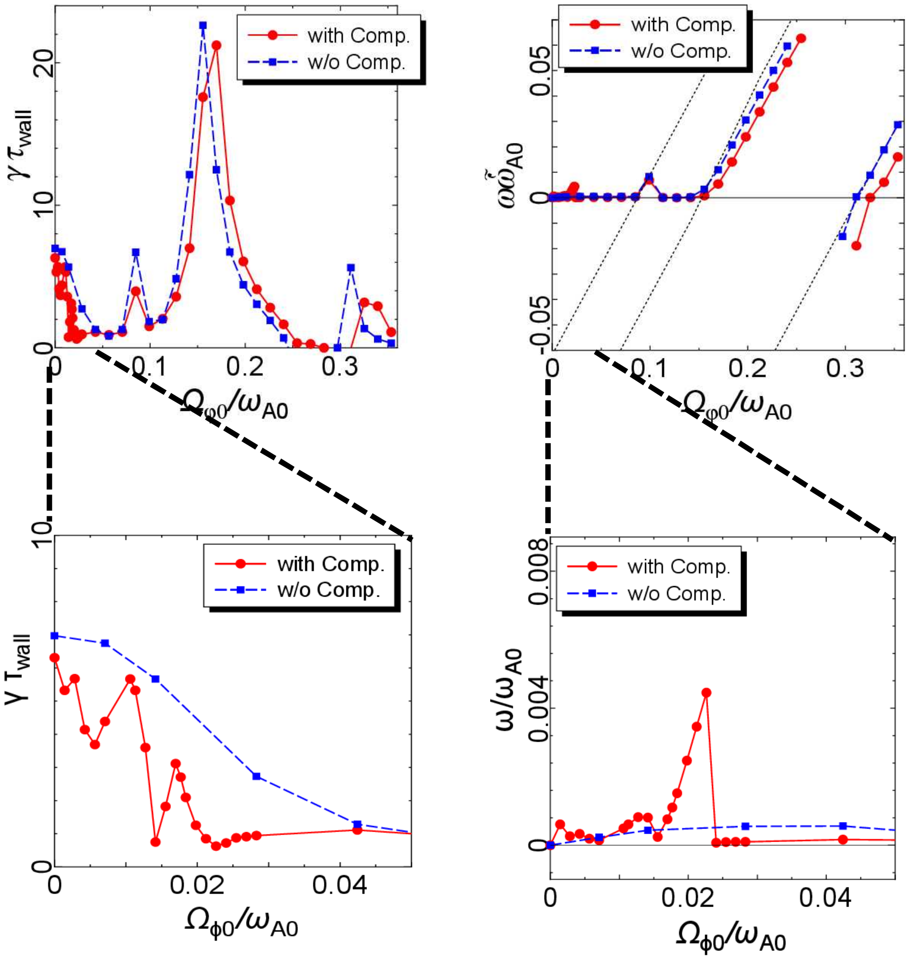
Mode frequencies are Doppler-shifted when rotation frequency passes the eigenmode frequency.

Mode resonance is essential for this RWM destabilization



Plasma compression increases the number of resonant mode

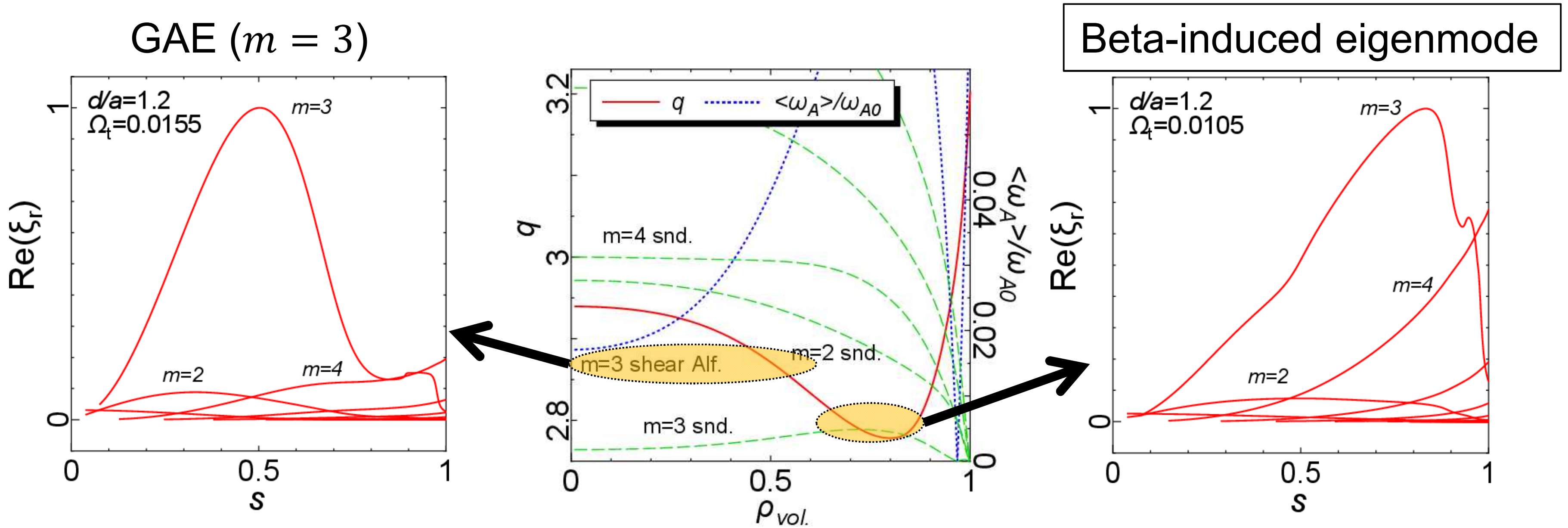
In the compressible case with $\Gamma = 5/3$, the RWM destabilization is also observed. However, we found the following difference from the result in the incompressible case.



With plasma compression, RWM can become stable with low Ω_t, but new resonant modes appears.
One of these new resonant modes shows clear Doppler-shift.

What causes such differences?

Sound wave continuum will be responsible for a new mode

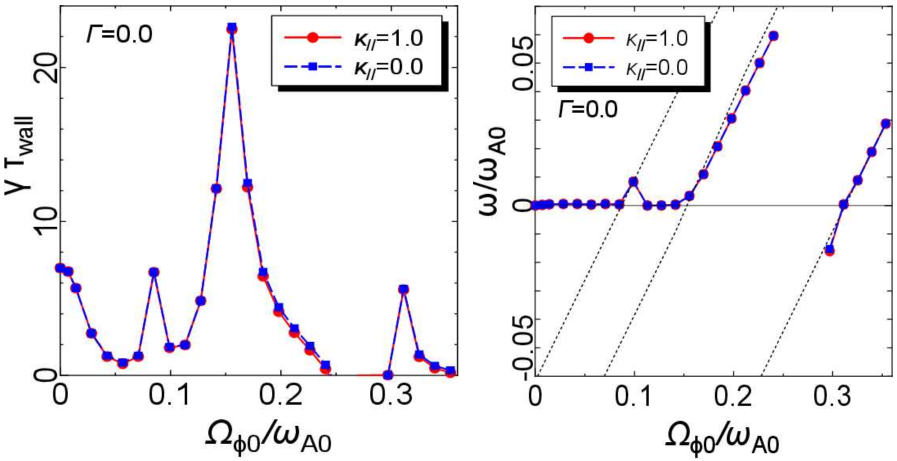


One of the new modes exists above $m = 3$ sound wave continuum.
⇒ The mode would be beta-induced reversed shear eigenmode.

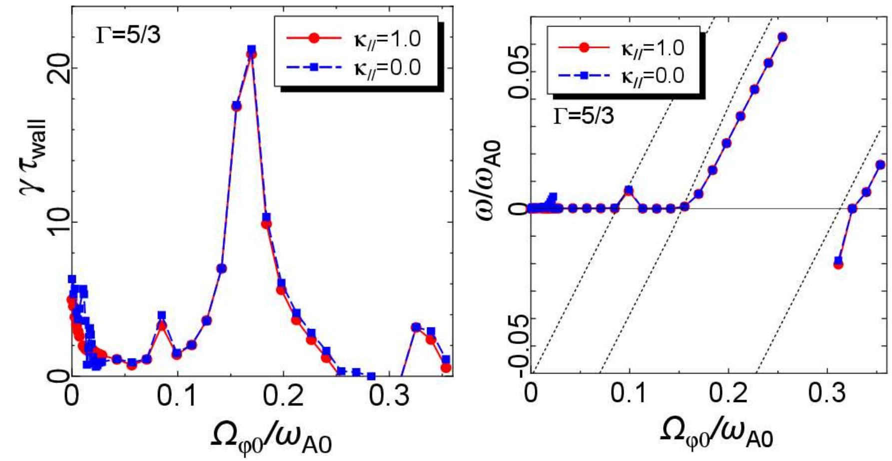
The other mode exists below $m = 3$ shear Alfven continuum with flat q profile near axis.
⇒ The mode would be AE (GAE?). This mode appears due to stabilizing original RWM in the incompressible case.

Sound wave damping stabilizes low-frequency resonant modes

Incompressible case



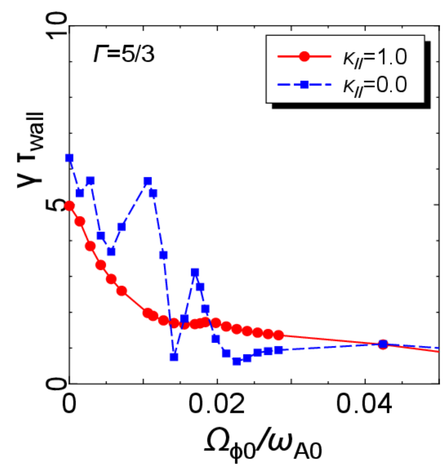
Compressible case



In the incompressible case, sound wave damping has no effect on stability (consistent with theory).

Also, in the compressible case, stability of the unstable mode coupling with RSAE/kink/GAE changes little.

The main different caused by sound wave damping is the stabilization of the mode in low-rotation frequency. (this would be consistent with theoretical prediction)



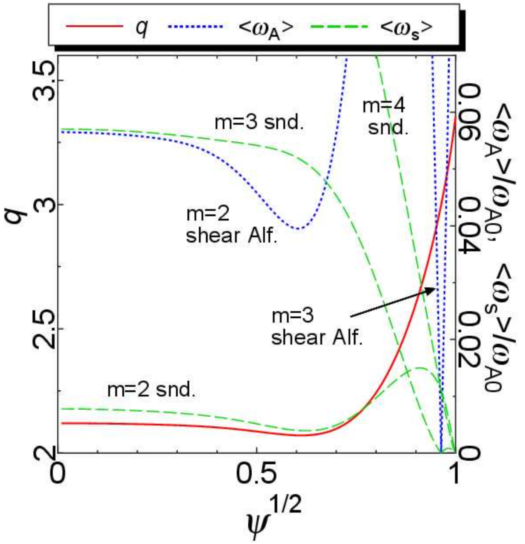
Equilibrium with low-freq. local minima/maxima of continua

In the simplest flat q equilibrium, MHD instability robustly appears. Hereafter, we make the equilibrium more experiment-relevant.

2nd step: q profile changes to weak reversed shear circular one

Plasma parameters

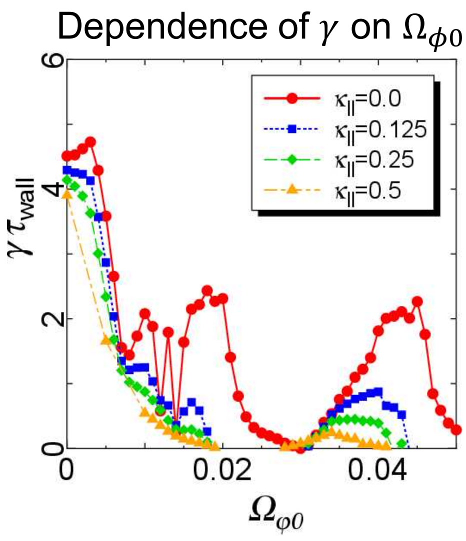
$$\beta_p = 0.7$$
$$B_T = 1.0[T]$$
$$I_p = 0.6[MA]$$
$$q_{min} = 2.07$$
$$\frac{d}{a} = 1.3$$

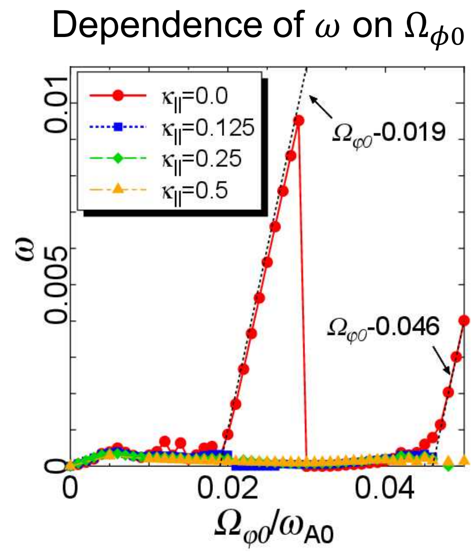


In this plasma, there are several local minima/maxima of sound wave and shear Alfven continua in $\omega < 0.05\omega_{A0}$. (conventional tokamaks usually have $\Omega_t \leq 0.05\omega_{A0}$)

- Plasma rotation profile is kept as rigid toroidal rotation.
- Since ion Landau damping effect becomes weak in tokamaks when rotation frequency is small compared with the sound wave one [Mikhailovskii PPR1995, Bondeson PoP1996], $\kappa_\parallel$ changes from 0 to 0.5.

Resonant modes are excited by coupling with low-frequency eigenmodes



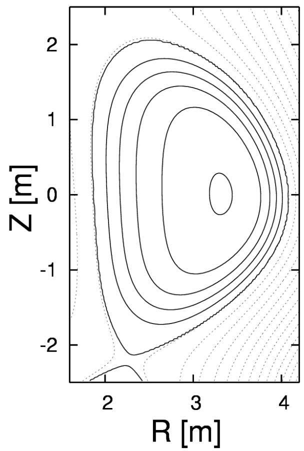


- With ideal MHD model, we found several local maxima of $\Omega_{\phi 0}$ dependence of γ, and two modes are clearly Doppler-shifted. (From continuum spectra, we can speculate them as the excited beta-induced eigenmode ($m = 2$) and RSAE ($m = 3$))
- Rotation with ion Landau damping effect stabilized RWM, but the destabilized mode remains unstable in $0.03 \leq \Omega_{\phi 0} \leq 0.044$.
- Negative energy ideal modes are strongly stabilized by ion Landau damping (mode frequency doesn't show clearly Doppler-shift when $\kappa_{||} \neq 0$).

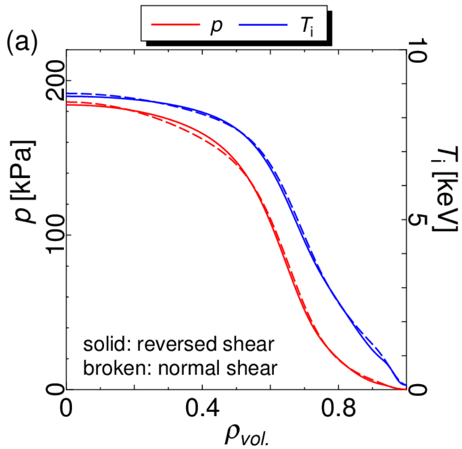
RWM destabilization by rotation

Last step: D-shape, and toroidal rotation with shear

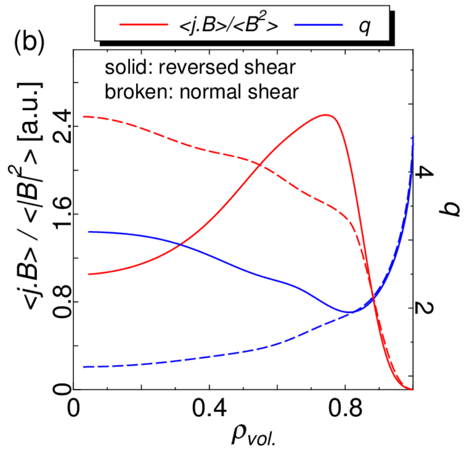
Contours of ψ



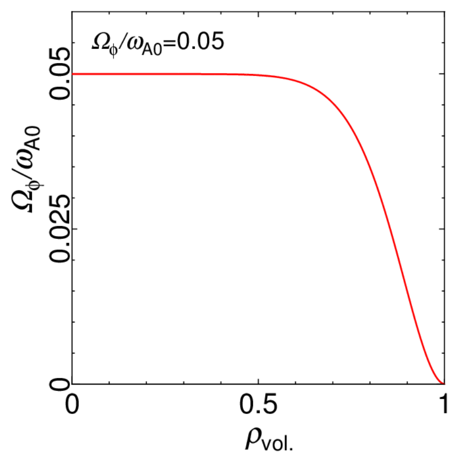
Profiles of p and T_i



Profiles of $j_{\parallel}$ and q



$\beta_N = 5.0, I_{\mathrm{p}} = 2.9[\mathrm{MA}]$
$q_0 = 3.1, q_{\mathrm{a}} = 4.6$
$(q_{min} = 2.03)$



Ideal wall position required for marginal stability is the same in both normal shear and reversed shear plasmas ($d/a|_{ideal} = 1.43$).

Rotation profile is given artificially as

$$\Omega_\phi = \Omega_{\phi 0}(1 - \psi^5)^2 \omega_{A0}.$$

$\Omega_{\phi 0}$: rotation freq. on axis

ω_{A0}: Shear Alfven freq. on axis