Hall and gyro-viscous effects to the growth of the Rayleigh-Taylor instability

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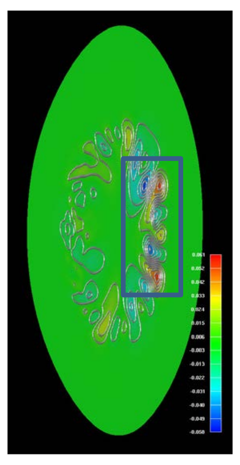
Background

- Numerical simulations using the single-fluid MHD model have been extensively carried out. [1]
- The single-fluid model ignores small scale effects such as Hall effect, Finite Larmor Radius (FLR) effect and so on.
- The single-fluid model can be insufficient for high wave number ballooning modes. Extended MHD model [2] may be reasonable for high wave number instability.
- For simplicity, a simple Rayleigh-Taylor (RT) instability is considered.



Model and Method

- Extended MHD equations (the Hall term and gyro-viscosity are added.)

$$\frac{\partial \rho}{\partial t} = -\nabla\cdot(\rho\mathbf{v}), \quad (1)$$

$$\frac{\partial}{\partial t}(\rho\mathbf{v}) = -\nabla\cdot\left[\rho\mathbf{v}\mathbf{v} + \mathbf{I}\left(p+\frac{B^2}{2}\right) - \mathbf{B}\mathbf{B} + \underline{\delta\mathbf{\Pi}_i}\right] + \rho\mathbf{g}, \quad (2)$$

$$\frac{\partial e}{\partial t} = -\nabla\cdot\left[\mathbf{v}(e+p) \underline{- \mathbf{v}\cdot\delta\mathbf{\Pi}_i}\right], \quad (3)$$

→ gyro-viscosity

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla\cdot(\mathbf{v}\mathbf{B} - \mathbf{B}\mathbf{v}) - \nabla\times\left[\frac{\epsilon}{\rho}(\mathbf{J}\times\mathbf{B} - \nabla p_e)\right]. \quad (4)$$

$$e = \frac{\rho \mathbf{v}^2}{2} + \frac{p}{\gamma - 1}$$

↓ Hall term

ϵ : Hall parameter
δ : gyro-viscous coefficient

- 2D approximations in a 2D slab $u_z = 0, \partial/\partial z \to 0$ are applied in eqs.(1)-(4).
- Electromagnetic fields $\mathbf{B}, \mathbf{E}, \mathbf{J}$ are expressed by three (x, y, z) components.

$$J_x = \frac{\partial B_z}{\partial y}, J_y = \frac{\partial B_z}{\partial x}, J_z = \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}$$

- The gyro-viscosity is given as

$$(\Pi_i)_{xx} = -(\Pi_i)_{yy} = -p_i\left(\frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y}\right), \quad (5)$$

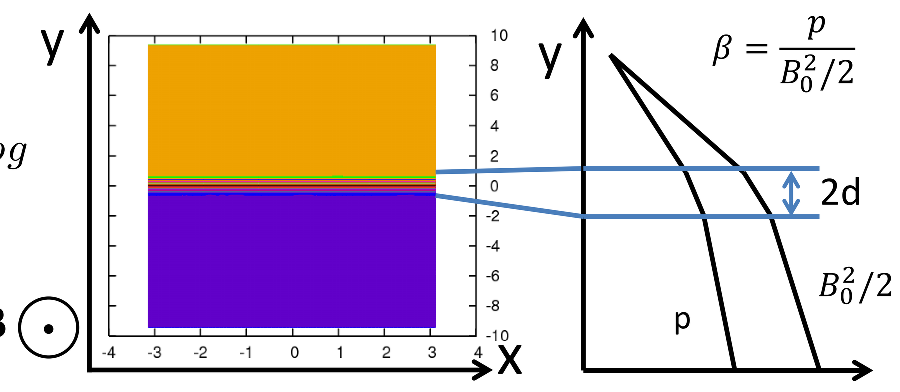
$$(\Pi_i)_{xy} = (\Pi_i)_{yx} = p_i\left(\frac{\partial u_x}{\partial x} - \frac{\partial u_y}{\partial y}\right). \quad (6)$$

g : gravitational acceleration, γ : ratio of the specific heats
total pressure : $p = p_i + p_e = (\alpha+1)p_e, \alpha = p_i/p_e$.
p_i, p_e: ion and electron pressure

Initial equilibrium and numerical method

- Equilibrium

$$\nabla\left(p_0 + \frac{B_0^2}{2}\right) = -\rho g$$



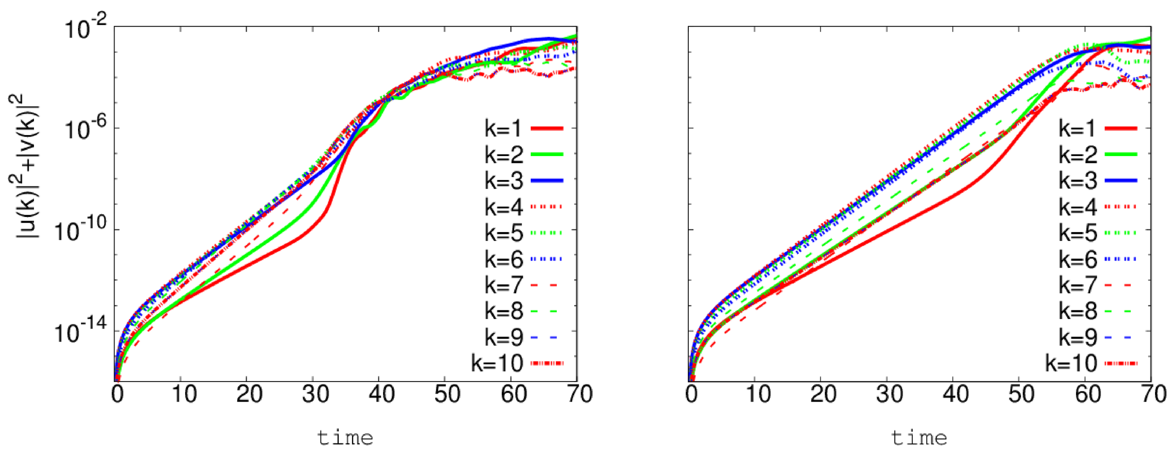
- Numerical method
→ • Space derivative : 4th order central difference
  • Time evolution : 4th order Runge-Kutta-Gill (RKG)
- System size : $-\pi \le x \le \pi$, $-3\pi \le y \le 3\pi$
- Boundary condition : periodic ($x = \pm\pi$) , $\partial/\partial y \to 0$ ($y = \pm 3\pi$)
- Resolution : $(N_x, N_y) = (1024, 4086)$
- density ratio : $\rho_2/\rho_1 = 2.0$
- β = 10%, density jump width : 1.0, p_i/p_e = 1.0

Reference

[1] H. Miura and N. Nakajima: Nucl. Fusion **50** (2010) 054006.
[2] P. Zhu et al: Phys. Rev. Lett. **101** (2008) 085005.

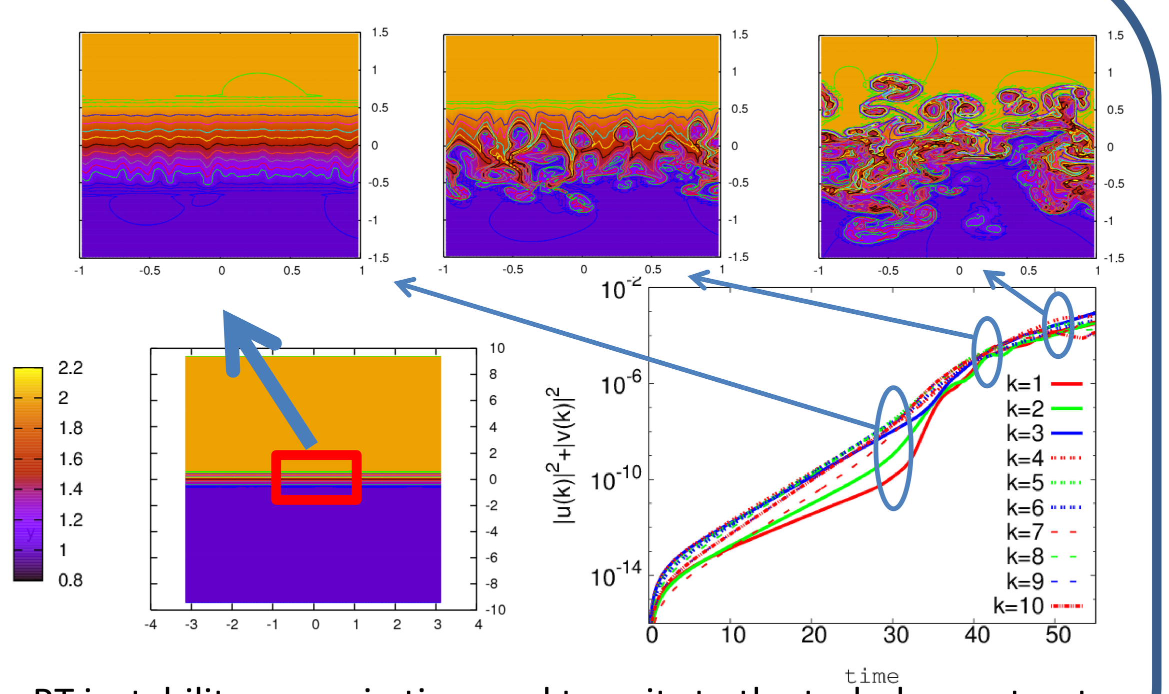
Growth of the Fourier modes

- Random perturbations are added simultaneously in the initial equilibrium profile.



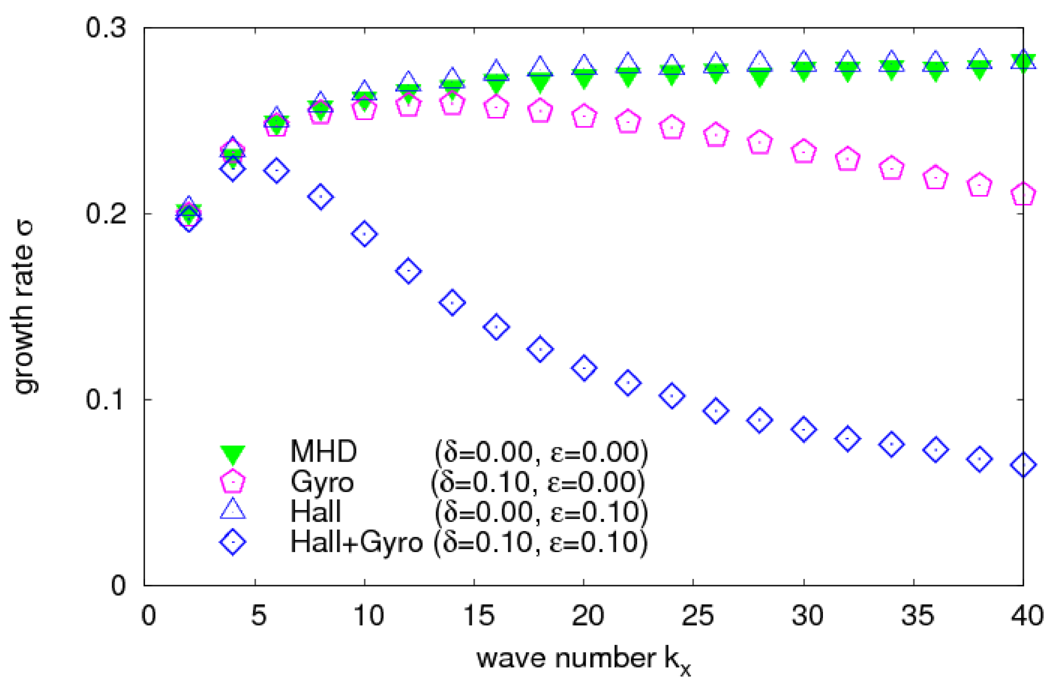
MHD ($\delta = 0.00, \epsilon = 0.00$) Hall+Gyro ($\delta = 0.10, \epsilon = 0.10$)

Time evolution of the R-T instability



- RT instability grows in time and transits to the turbulence structure.

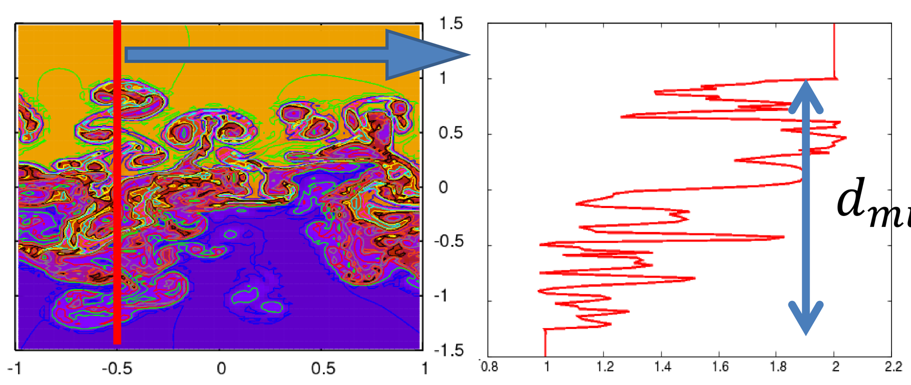
Growth rate : suppression of the higher modes



- Linear growth rate is evaluated by the gradient of the integrated kinetic energy.
- In this parameter, the gyro-viscosity term reduces the growth rate of the high wave number modes, while the Hall term slightly destabilizes them.
- When the Hall term and the gyro-viscosity are added simultaneously, growth rate of the high wave number mode is strongly reduced.

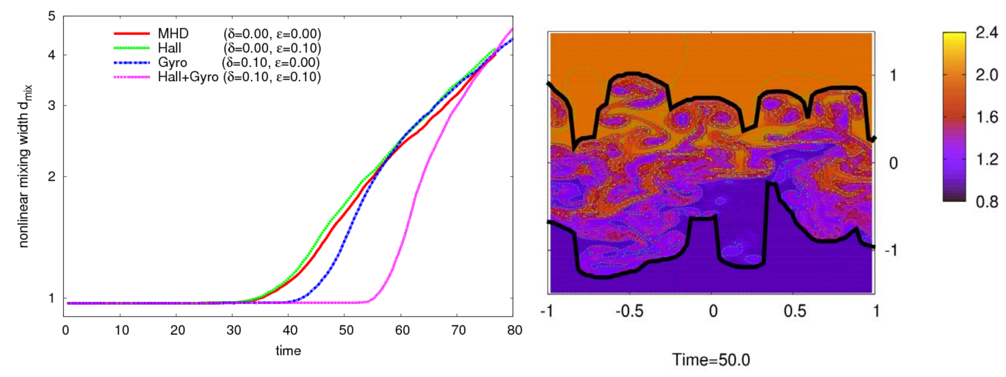
Nonlinear stage : Mixing width I

- Mixing width is used as an index of the enhancement of the R-T instability in the nonlinear stage.
- Since mixing width is different at each x coordinate, mixing width is averaged out in the x direction.



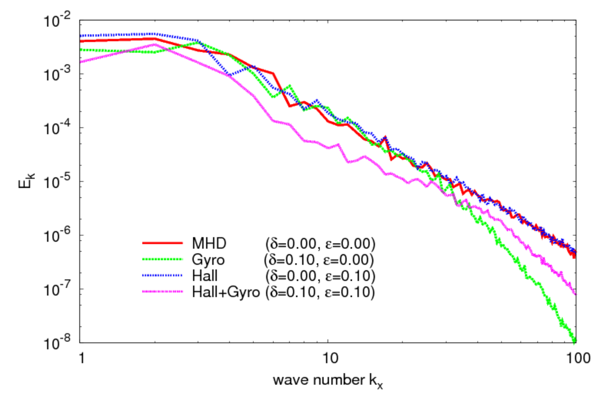
$$d_{mix} \equiv \frac{1}{N}\sum d_{mix,x_i}$$

Nonlinear stage : Mixing width II



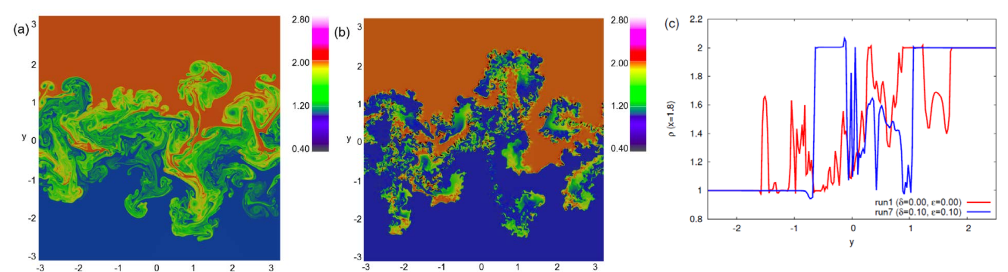
- Mixing width remains constant in the linear stage and rapidly increases in the nonlinear stage.
- In the Hall case, the mixing width slightly increases compared with MHD case.
- In the Gyro and Hall+Gyro cases, time scale of the linear stage becomes long. However, the mixing width increases rapidly compared with the other cases.
- These results imply that the suppression of the linear growth rate does not always lead to the reduction of the mixing width.

Nonlinear stage : Energy spectrum



- In the Hall+Gyro case, energy spectrum is lower than Gyro case ($30 < k_x$) and larger than Gyro case ($30 > k_x$).
- In the Hall +Gyro case, Hall term may reduce the energy of the low wave number modes.

Nonlinear stage : the mass density changes sharply



- While in the MHD case, the mass density changes smoothly, in the Hall+Gyro case, the mass density changes sharply.
- The mass density suddenly changes from ρ_1 to ρ_2 .
→ May be due to the combination of the Hall term and the gyro-viscosity.

Summary

- The effect of the Hall term and the gyro-viscosity to the RT instability is studied by the nonlinear extended MHD simulation.
- The Hall term slightly increases the growth rate, while the gyro-viscosity reduces the growth rate of the high wave number mode.
- When the Hall term and the gyro-viscosity are added simultaneously, the linear growth rate of the high wave number mode is strongly reduced compared with other cases.
- The Hall term slightly increases the mixing width in the nonlinear stage compared with MHD result.
- Nonlinear mixing width is rapidly increased in the Gyro and Hall+Gyro cases.
- In our simulations, reduction of the linear growth rate does not lead to the reduction of the nonlinear mixing width.

Future plan

- Extended MHD code that calculates near the edge region is now under developing.
- The geometry will be extended to a 3D torus system to analyze the evolution of the ELMs.

2D annular torus → 3D annular torus