Impact of Centrifugal Modification of Magnetohydrodynamic Equilibrium on Resistive Wall Mode Stability

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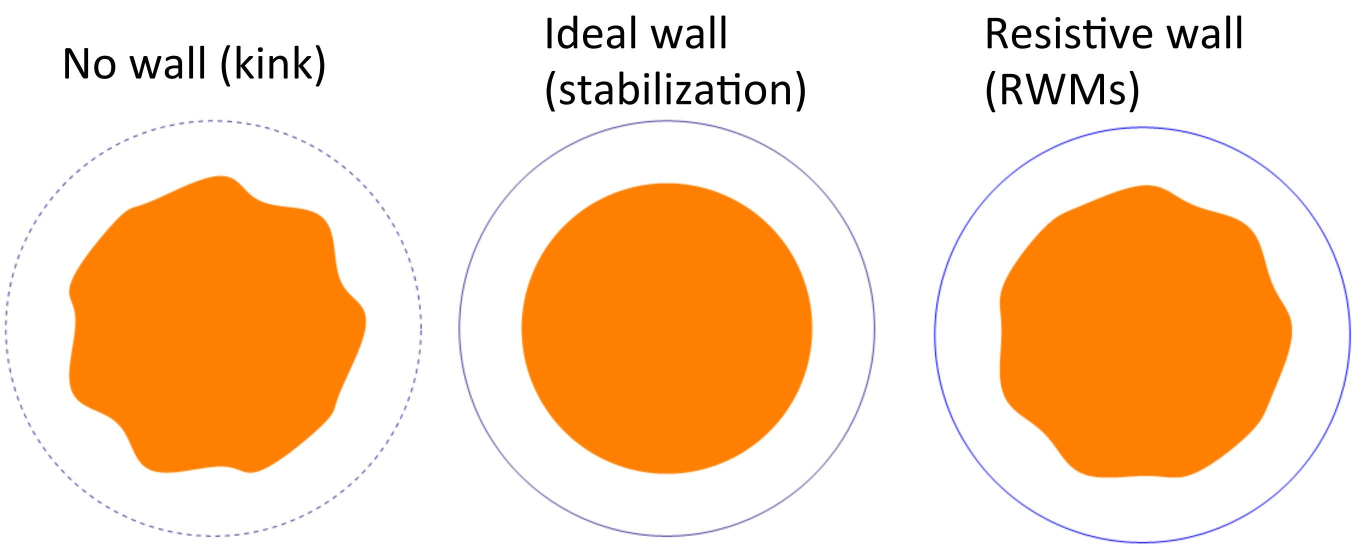
Summary

- ✓ RWMs in self-consistent (including rotational modification of the Grad-Shafranov equation) equilibria have been numerically investigated.
- ✓ Compared with conventional (using the static Grad-Shafranov equation) equilibria, in the self-consistent equilbria,
  - ✓ RWM growth rates are reduced for a wide parameter range of β_N(=3~5), rotation ($M^2 <\sim 0.15$) and wall location.
  - ✓ The stable window is enlarged and shifted.
  - ✓ Stable windows can exist even if the conventional equilibria have no window.
- ✓ The modification of equilibrium profile, not of eigenfunction, is essential to stabilization in self-consistent equilibria.
- ✓ In self-consistent equilibria, reduction of destabilizing energy δW_{uhp} and δW_{uhc} is essential to stabilization.

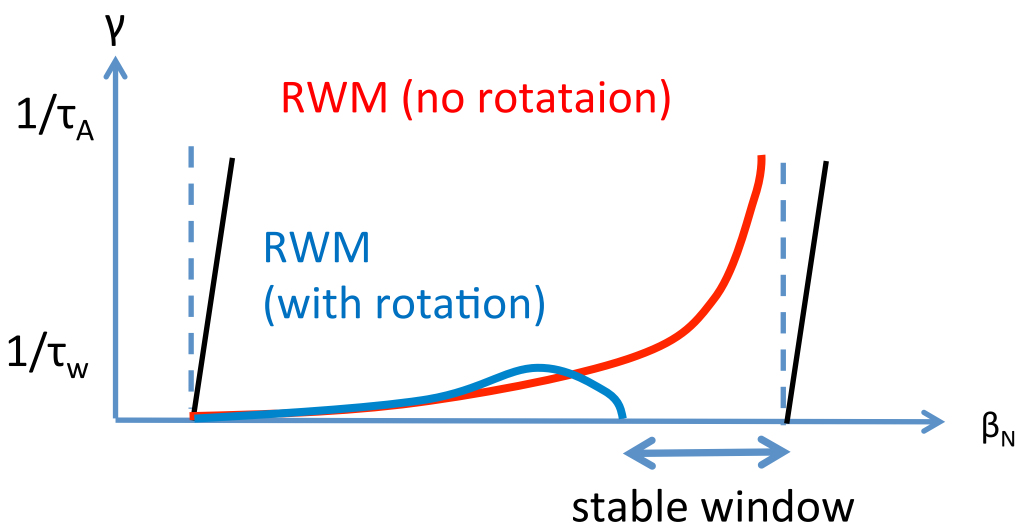
What is Resistive Wall Modes (RWMs)?

- ✓ RWMs originates from external kink modes ($\gamma\tau_A$~1).
- ✓ Ideal walls stabilize external kink modes.
- ✓ Resistive walls slow down external kink modes to timescale of eddy current decay ($\gamma\tau_w$~1).



Why RWM? How to stabilize RWM?

- ✓ Stabilization of RWMs is a necessary condition for operation of advanced tokamaks aiming at steady-state high-β_N plasma confinement such as JT-60SA.
- ✓ Many theoretical/experimental researches show rotational stabilization of RWMs.



As a basis of quantitative RWM study, we need to develop a numerical code for RWM in realistic tokamak geometry including plasma rotational effects.

RWM codes in tokamak geometry

- ✓ MARS-F (Chu PoP05), MARS-K (Liu NF09), CarMa (Liu PoP09)
  - ✓ Linearized resistive MHD, "perturbative" toroidal rotation, kinetic effects, 3D wall, feedback.
- ✓ NMA (Chu NF03)
  - ✓ Linearized ideal MHD without rotation, feedback.
- ✓ MISK (Berkery PRL11)
  - ✓ Linearized ideal MHD without rotation, kinetic effects
- ✓ VALEN (Bialek PoP01)
  - ✓ Linearized ideal MHD without rotation, 3D wall, and feedback
- ✓ **MINERVA(Aiba CPC09) with "RWMaC" modules**
  - ✓ We develop a new RWM code. It has some advantages :
  - ✓ (1) perturbative poloidal rotation (Aiba PoP11)
  - ✓ (2) **centrifugal modification of MHD equilibrium by plasma toroidal rotation**
  - ✓ (3) initial value approach

Rotational modification of MHD equilibrium

Under isothermal condition T=T(ψ), existence of toroidal rotation generalizes the Grad-Shafranov equation as (e.g. Zehrfeld NF72)

$$\begin{cases} \Delta^{*}\psi = -F\dfrac{dF}{d\psi} - \mu_0 R^2 \left.\dfrac{\partial p}{\partial \psi}\right|_R \\ p(\psi, R) = p_0(\psi)\exp\left[M^2(\psi)\left(\dfrac{R^2}{R_0^2} - 1\right)\right] \end{cases} \quad (1)$$

Definition of two types of equilibria used in this poster

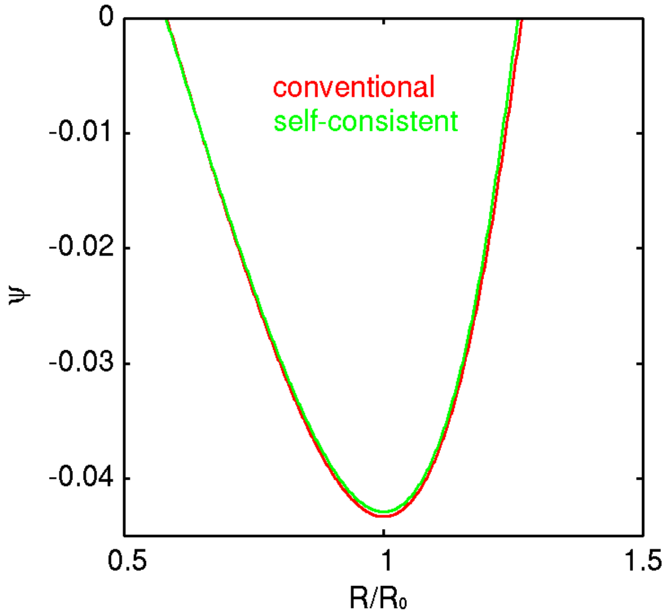
We call the solution to (1) as **"self-consistent equilibrium."**

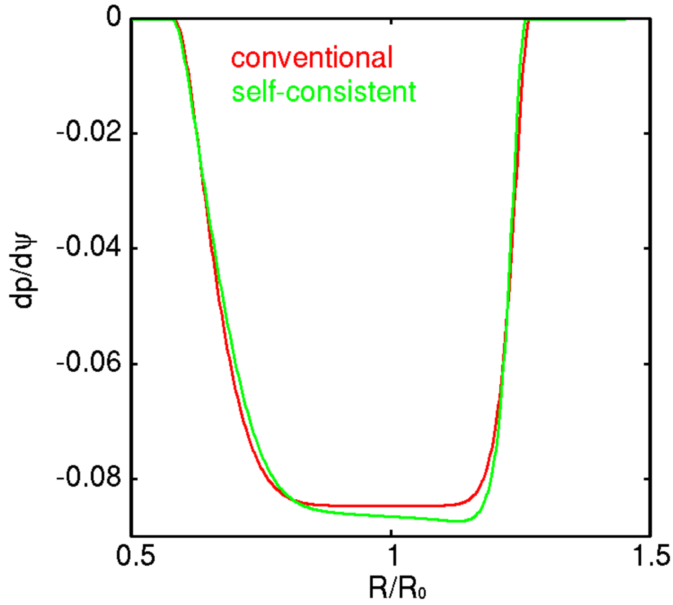
Conventional studies approximate $p \sim p_0$ due to the smallness of M^2, i.e., the equilibrium is static.

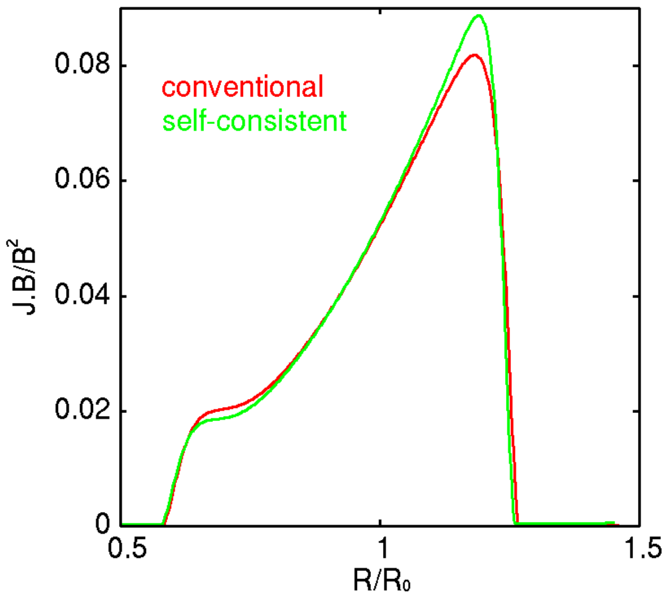
We call this approximated solution as **"conventional equilibrium."**

Rotational modification of MHD equilibrium (cont.)

Examples for "conventional" and "self-consistent" equilibria with β_N~2.83 and M^2=0.1







Note : Pressure gradient and parallel current are affected by self-consistent inclusion of rotation compared with ψ.

Formalism for linear dynamics of RWMs – vacuum and resistive wall dynamics

Ampère's law across the resistive wall

$$\chi^{(w+)}(\theta,\phi,t)-\chi^{(w-)}(\theta,\phi,t)=\mu_0\kappa \qquad (2)$$

Faraday's and Ohm's laws across the resistive wall

$$\frac{\Delta|\nabla s|}{\eta}\frac{\partial\widetilde{B}^{(n)}}{\partial t}=-\nabla\cdot\left(|\nabla s|^2\nabla_\perp\kappa\right) \qquad (3)$$

Quadratic form from (2) and (3) yields energy balance

$$\delta W_{IV}+\delta W_{OV}+\frac{1}{2\mu_0}\int_{S_p}\chi^{(p+)}\mathbf{Q}_e^*\cdot\hat{n}dS+\delta D_w=0 \qquad (4)$$

plasma response

vacuum magnetic energy $\delta W_{IV(OV)}=\frac{1}{2\mu_0}\int_{IV(OV)}\left|\nabla\chi^{\pm}\right|^2 d\tau$

energy dissipation in resistive wall $\delta D_w=\frac{1}{2}\int_{\text{wall}}\kappa\mathbf{B}\cdot d\mathbf{S}$

Formalism for linear dynamics of RWMs – linear plasma response

Plasma is governed by the Frieman-Rotenberg equation (Frieman RMP60), which is the linearized ideal MHD with equilibrium rotation.

$$\rho\partial_t^2\boldsymbol{\xi} + 2\rho(\mathbf{u}\cdot\nabla)\partial_t\boldsymbol{\xi} = \left(F_s + F_d\right)\boldsymbol{\xi} \qquad (5)$$

static and dynamic force operator

Linear plasma response of the Fireman-Rotenberg equation reads

$$\frac{1}{2\mu_0}\int_{S_p}\chi^{(p+)}\mathbf{Q}_e^*\cdot\hat{n}dS = \delta K + 2\delta W_c + \delta W_p \qquad (6)$$

Kinetic energy : $\delta K = \frac{1}{2}\int\boldsymbol{\xi}^*\cdot\rho\partial_t^2\boldsymbol{\xi}d\tau$

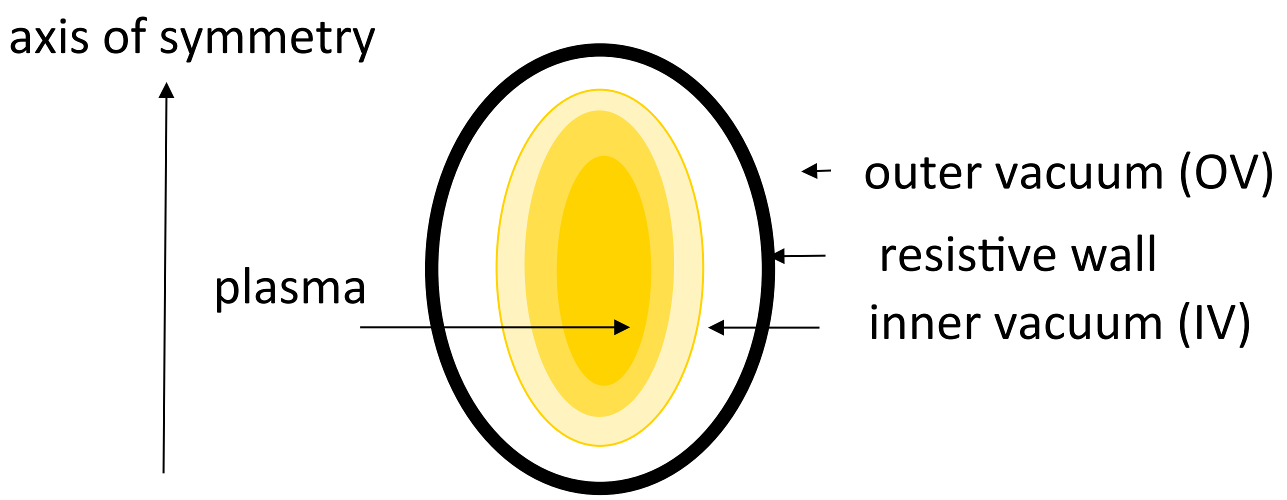
convective energy : $\delta W_c = \frac{1}{2}\int\boldsymbol{\xi}^*\cdot\rho\left(\mathbf{u}\cdot\nabla\right)\partial_t\boldsymbol{\xi}d\tau$

Potential energy including equilibrium rotation

$$\delta W_p = \frac{1}{2}\int\boldsymbol{\xi}^*\cdot\left[\frac{|\mathbf{Q}|^2}{\mu_0} + \mathbf{J}\cdot\left(\boldsymbol{\xi}^*\times\mathbf{Q}\right) + \left(\boldsymbol{\xi}\cdot\nabla p\right)\nabla\cdot\boldsymbol{\xi}^* + \Gamma p|\nabla\cdot\boldsymbol{\xi}|^2 - \rho\boldsymbol{\xi}\left(\mathbf{u}\cdot\nabla\right)\mathbf{u} + \rho\mathbf{u}\left(\mathbf{u}\cdot\nabla\right)\boldsymbol{\xi}\right]d\tau \qquad (7)$$

Note : "Conventional equilibrium" approach introduces equilibrium rotation **u** in (6). The equilibrium quantities such as **B**, p, and ρ include the modification induced by toroidal rotation, which is based on the solution of "static" equilibrium.

Formalism for linear dynamics of RWMs – energy balance



(4) and (6) yield energy balance in plasma-wall-vacuum system.

plasma | vacuum | wall

$$\delta K + 2\delta W_c + \delta W_p + \delta W_{OV} + \delta W_{IV} + \delta D_W = 0 \quad (8)$$

kinetic energy; convective energy; potential energy with rotation; vacuum magnetic energy in IV/OV; energy dissipation in resistive wall

RWM dynamics is governed by the balance among these energy sources (sinks).

"RWMaC" modules compute $\delta W_{IV(OV)}$ and δD_w

To solve (8) we have developed "RWMaC" modules to compute $\delta W_{IV(OV)}$ and δD_w, and have implemented them into MINERVA (Aiba CCP09) that computes δK, δW_p, and δW_c.

Inner vacuum and outer vacuum : $\delta W_{IV(OV)}$
Governing equation : Laplace equation for χ
(magnetic scalar potential $\mathbf{B}=\nabla\chi$)
Numerical scheme : hybrid FEM for IV
hybrid FEM or Green's function
method for OV

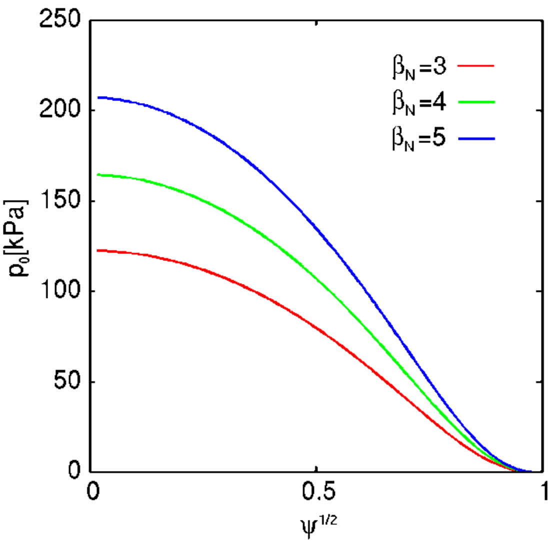
Resistive wall : δD_w
Governing equation : diffusion equation for κ
[current potential $\mathbf{J}=(\nabla s\times\nabla\kappa)\delta(s-s_w)$]
Numerical scheme : hybrid FEM

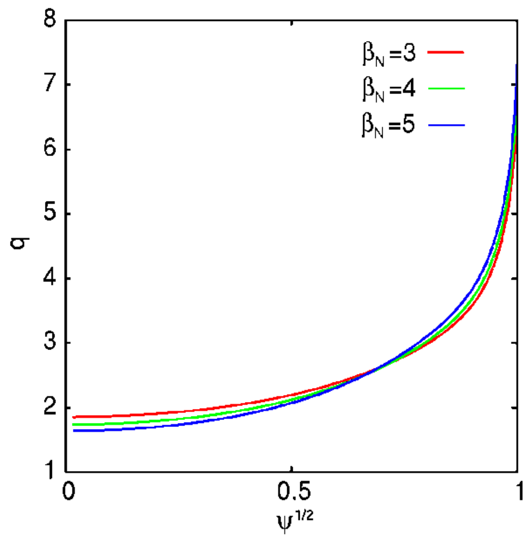
Boundary conditions on resistive wall and plasma surface :
Continuity of normal magnetic field + natural boundary condition

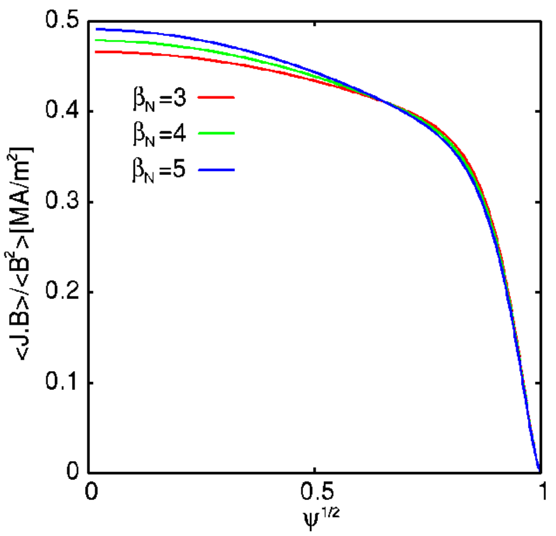
RWMs in self-consistent equilibria – high-β_N equilibria for JT-60SA

By MINERVA/RWMaC, we can study the RWMs in self-consistent equilibria.

We consider high-β_N ($2.8<\beta_N<5.5$) equilibria with fixing D-shape of plasma surface (κ=1.91 and δ =0.5), toroidal magnetic field B_0=1.7T, and plasma current I_p=2.3MA, which are typical parameters for advanced plasma designed for JT-60SA.





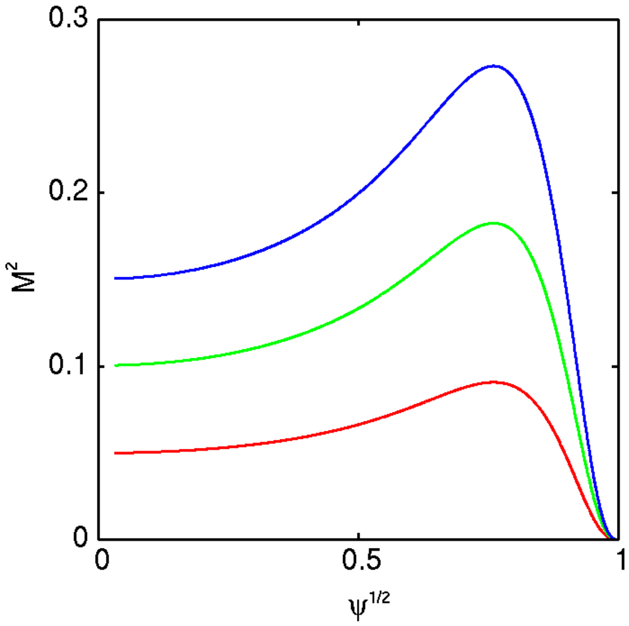


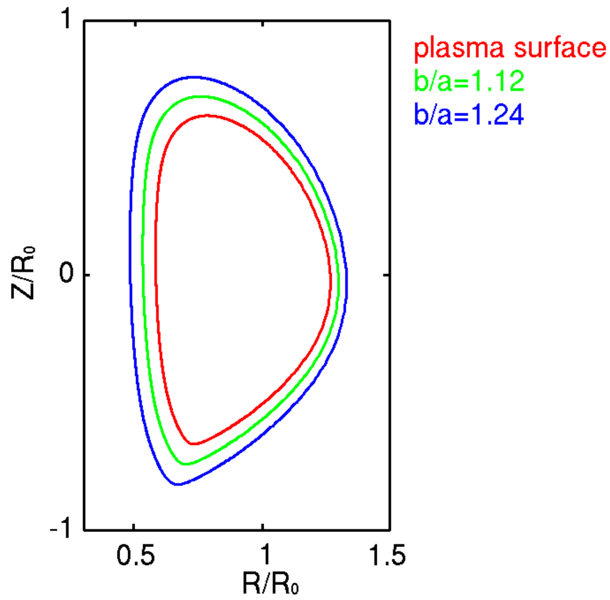
β_N is increased by scaling p_0 with keeping the almost same profiles of safety factor and parallel current.

RWMs in self-consistent equilibria – wall location rotation profiles

Rotation profiles is characterized by $\Omega(\psi)=\Omega_0[1-(\psi/\psi_{sur})^5]^2$. By changing Ω_0 and fixing temperature at magnetic axis, we consider 3 cases of squared Mach number rotation as M^2=0.05,0.1, and 0.15 which are relevant to low-aspect ratio tokamaks.

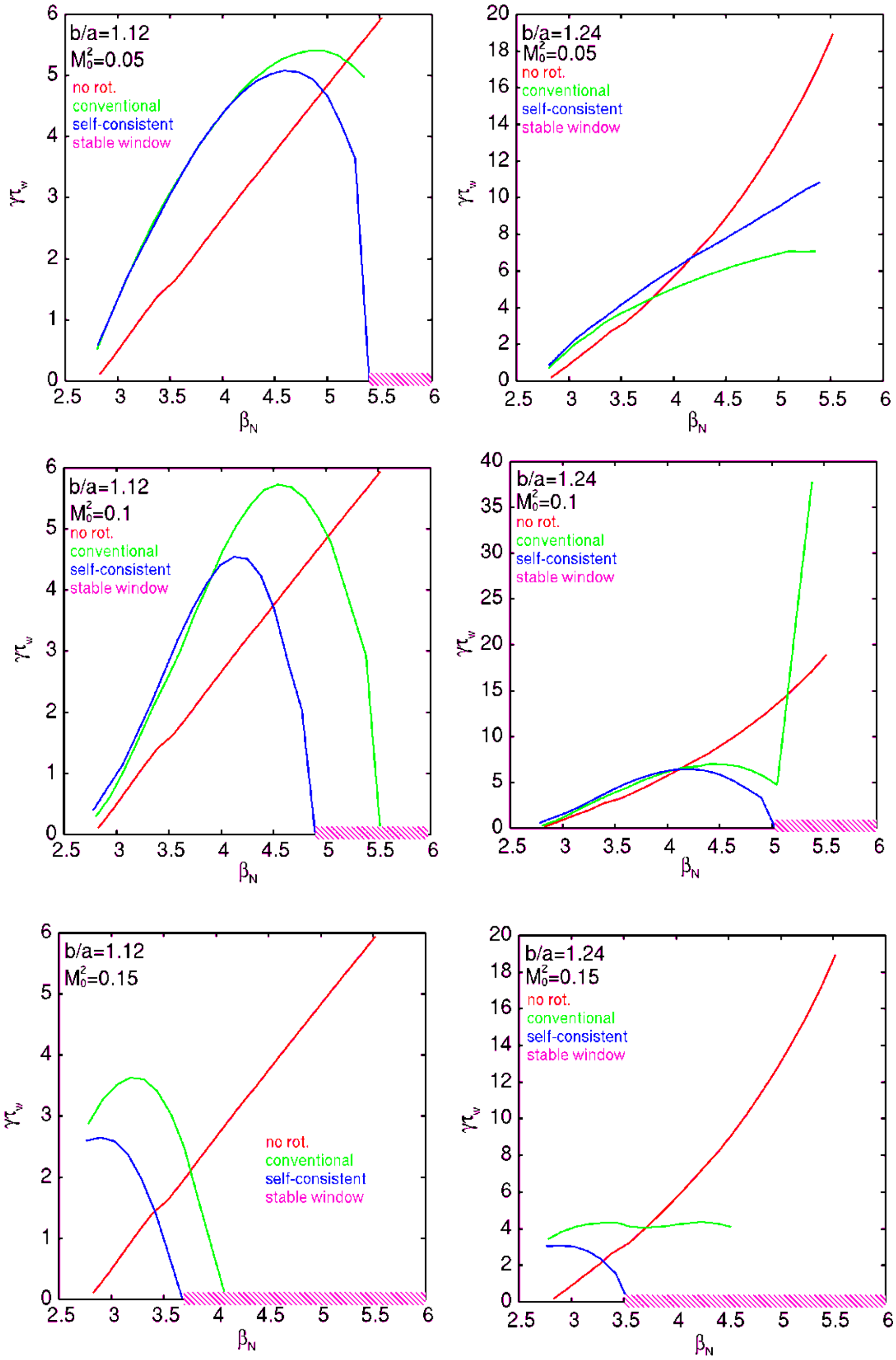
We use two wall locations b/a=1.12 and 1.24.





RWM stabilization in self-consistent equilibria – scan by β_N, rotation, and wall location

Normalized RWM growth rates without rotation, in conventional equilibrium, and self-consistent equilibrium as functions of β_N



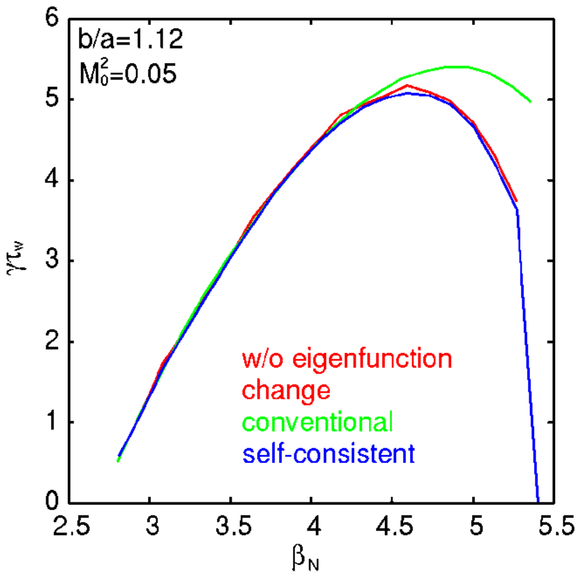
Note on scan by β_N ,rotation, and wall location

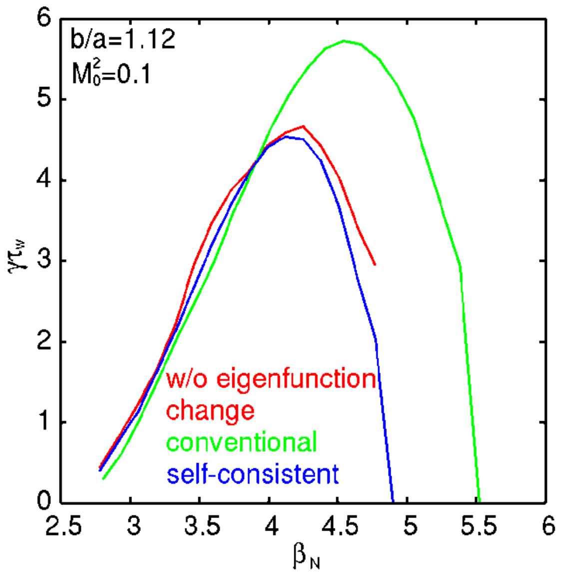
- ✓ RWM growth rates are reduced in a wide range of wall location and β_N.
- ✓ Some cases (b/a=1.12 and M^2=0.1, 0.15) show that
  - ✓ the self-consistent equilibrium has an extended stable window.
  - ✓ the location of stable window is shifted.
- ✓ Other cases (b/a=1.12 and M^2=0.05, b/a=1.24 and M^2=0.1, 0.15) show that
  - ✓ The self-consistent equilibrium has a stable window even if the conventional equilibrium has no window.

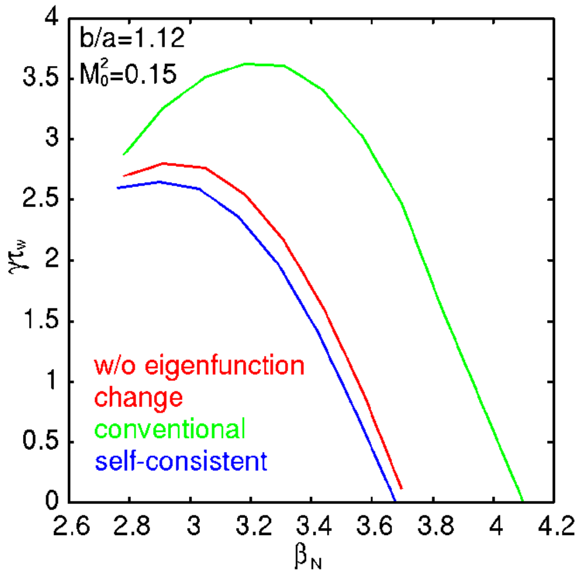
In self-consistent equilibria, RWMs are stabilized by equilibrium change self-consistently introduced in the equilibrium.

Modification of equilibrium is essential to RWM stability – eigenfunction modification is not

Rotation modifies eigenfunction as well as equilibrium.







RWM problem reads $A\boldsymbol{\xi}=\lambda B\boldsymbol{\xi}$. Defining Δ by $\Delta f=f_s-f_c$ where s (c) indicates self-consistent (conventional), the problem reads

$$(A_c - \lambda_c B_c)\Delta\boldsymbol{\xi} + B_c\Delta\left(B^{-1}A\right)\boldsymbol{\xi}_s = \Delta\lambda B_c\boldsymbol{\xi}_s$$

eigenfunction change | equilibrium change | eigenvalue change

Reduction of pressure and current driven terms is essential to RWM stabilization

From energy balance (8), we get

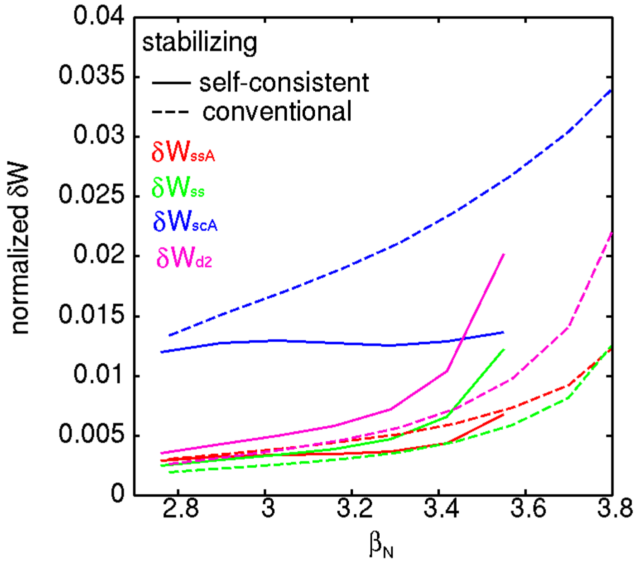
$$\gamma = \mathrm{Re}(\lambda) = -\frac{\delta W_p + \delta W_{IV} + \delta W_{OV}}{\left|\delta D_w + 2\delta W_c^2\right|^2}\,\delta D_w$$

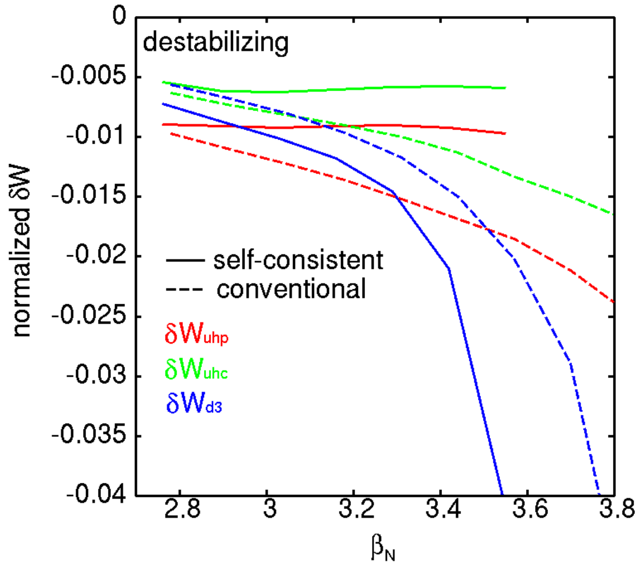
We decompose potential energy δW_p as

$$\delta W_p = \delta W_{ssA} + \delta W_{ss} + \delta W_{scA}$$: stabilizing

$$+\,\delta W_{uhp} + \delta W_{uhc}$$: destabilizing by pressure and current driven

$$+\,\delta W_{d1} + \delta W_{d2} + \delta W_{d3}$$:mode compression, shear, centrifugal effect due to rotation





In self-consistent equilibrium, we get smaller δW_{scA} and larger δW_{d3}. So it seems to lead to destabilization, however, reduction of destabilizing δW_{uhp} and δW_{uhc} leads to stabilization