Simulation of two-dimensional transport in tokamak plasmas

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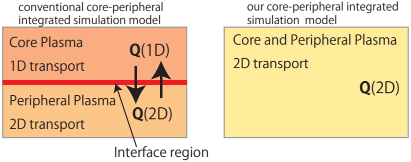
Background and motivation



- Conventional transport simulation of tokamak plasmas
    - In the core of a tokamak plasma transport phenomena have been usually described as one-dimensional problems.
    - In the peripheral SOL-divertor plasma, transport phenomena are described as two-dimensional problems

Recent remarkable progress in computational technology has made more consistent two-dimensional transport simulation of tokamak plasmas feasible.

To carry out two-dimensional transport analysis

- Transport model including poloidal-angle dependence is required.

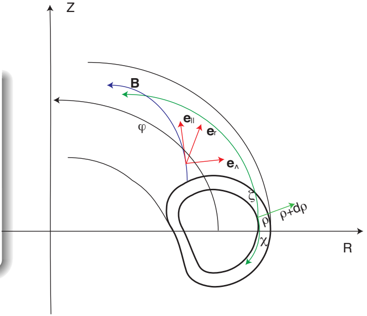
By employing 2D transport model over the entire plasma

- Analysis of the poloidal angle dependence of the heating efficiency will become available.
- Analysis of the poloidal-angle-dependent transient phenomena will become available.

We formulate an axisymmetric two-dimensional transport modeling which analyzes time evolution of plasmas over the entire tokamak.

Coordinates

Magnetic Flux Coordinates System (MFCS):
$(\xi_1^M, \xi_2^M, \xi_3^M) = (\rho, \chi, \zeta)$
Suitable description of MHD equilibrium configuration



Local Orthogonal Coordinate System (LOCS):
$(\xi_1^L, \xi_2^L, \xi_3^L) = (r, \wedge, \|)$
Simpler description on the behavior of magnetized plasmas

Neoclassical Transport Coordinate System (NTCS):
$(\xi_1^N, \xi_2^N, \xi_3^N) = (\rho, \|, \zeta)$
Good compatibility with neoclassical theory

Assumptions

- Toroidally axisymmetric plasmas
  - Quantities are independent of the toroidal angle variable
- Quantities related to MHD equilibrium depend only on the flux label
- Relaxation processes much slower than Alfvén time scale
- Weak time dependence of basis vectors
  - Time derivatives of basis vectors are small enough to be ignored
- Force balance in MHD time scale
  - Force balance in the radial direction is attained in the MHD time scale

Equations to be derived

Transport equations

- Equation for particle density: $n_{\mathrm{e}}(\rho,\chi)$, $n_{\mathrm{i}}(\rho,\chi)$
- Equation for momentum: $n_{\mathrm{e}}\boldsymbol{u}_{\mathrm{e}}(\rho,\chi)$, $n_{\mathrm{i}}\boldsymbol{u}_{\mathrm{i}}(\rho,\chi)$
- Equation for energy transport: $p_{\mathrm{e}}(\rho,\chi)$, $p_{\mathrm{i}}(\rho,\chi)$

Electromagnetic equations

- Poisson equation for electrostatic potential: $\phi(\rho)$
- Magnetic diffusion equation: $\iota(\rho)$
- Grad-Shafranov equation: $\psi(R,Z)$

Braginskii's equations

Two-fluid transport equation for cold plasma.

- Equation of continuity

$$\frac{\partial n_a}{\partial t} + \nabla \cdot (n_a \boldsymbol{u}_a) = S_a$$

- Equation of motion

$$\frac{\partial}{\partial t}(m_a n_a \boldsymbol{u}_a) = -\nabla \cdot (m_a n_a \boldsymbol{u}_a \boldsymbol{u}_a) - \nabla p_a - \nabla \cdot \overleftrightarrow{\pi}_a + e_a n_a (\boldsymbol{E} + \boldsymbol{u}_a \times \boldsymbol{B}) + \boldsymbol{R}_a + m_a S_{u_a} \boldsymbol{u}_a$$

- Equation for energy

$$\frac{3}{2}\frac{\partial p_a}{\partial t} + \nabla \cdot \left(\boldsymbol{q}_a + \frac{5}{2} p_a \boldsymbol{u}_a \right) = -\overleftrightarrow{\pi}_a : \nabla \boldsymbol{u}_a + \boldsymbol{u}_a \cdot \nabla p_a + Q_a + S_{p_a}$$

Formulation of transport equations

- Policy for the derivation of transport equations
  - For compatibility with the neoclassical transport theory, three components of vector quantities are represented by NTCS.
  - Spatial independent variables are expressed in MFCS
- Neoclassical viscosity tensor in LOCS

$$\overleftrightarrow{\pi}_a \equiv 3\frac{\boldsymbol{u}_a \cdot \nabla\chi}{\boldsymbol{B} \cdot \nabla\chi}\nabla_\parallel B \left(\frac{1}{3}\sum_{i=1}^{3}\boldsymbol{e}_{\xi_i^{\mathsf{L}}}\boldsymbol{e}_{\xi_i^{\mathsf{L}}} - \boldsymbol{e}_\parallel\boldsymbol{e}_\parallel\right)$$

- The equation for particle density

$$\frac{\partial n_a}{\partial t} + \sum_{i=1}^{3}\sum_{j=1}^{3}\frac{1}{\sqrt{g}}\frac{\partial}{\partial \xi_i^{\mathsf{M}}}(\sqrt{g}n_a\mathcal{T}_{ij}^{\mathsf{MN}}u_a^{\xi_j^{\mathsf{N}}}) = S_a$$

where $\mathcal{T}_{ij}^{\mathsf{MN}}$ is the transformation matrix from MFCS to NTCS.

- Equation for momentum

$$
\begin{aligned}
\frac{\partial}{\partial t}(m_a n_a \boldsymbol{u}_a) = & -\sum_{i=1}^{3} F_a^{\mathsf{kin},i} \boldsymbol{e}_{\xi_i^{\mathsf{M}}} - \nabla p_a \\
& - \frac{1}{3}\nabla N_a^{\mathsf{neo}} + \nabla_\parallel N_a^{\mathsf{neo}} - N_a^{\mathsf{neo}} \nabla_\parallel \ln B + N_a^{\mathsf{neo}} \boldsymbol{\kappa} \\
& + e_a n_a (\boldsymbol{E} + \boldsymbol{u}_a \times \boldsymbol{B}) + \boldsymbol{R}_a + m_a S_{u_a} \boldsymbol{u}_a
\end{aligned}
$$

where, $F_a^{\mathsf{kin},i}$ is the contravariant component of kinetic force in MFCS and N_a^{neo} the parallel coefficient of neoclassical viscosity.

The equation for momentum in each direction of NTCS is derived by taking the scalar product with $\boldsymbol{e}^{\xi_i^{\mathsf{N}}}$.

$$
\boldsymbol{e}^{\xi_1^{\mathsf{N}}} \equiv \nabla \rho, \quad \boldsymbol{e}^{\xi_2^{\mathsf{N}}} \equiv \boldsymbol{e}_\parallel, \quad \boldsymbol{e}^{\xi_3^{\mathsf{N}}} \equiv \nabla \zeta
$$

- Equation for momentum in radial direction

$$0 = -F_a^{\mathsf{kin},1} - \sum_{i=1}^{3} g^{1i}\frac{\partial p_a}{\partial \xi_i^{\mathsf{M}}} - \sum_{i=1}^{3}\frac{1}{3}g^{1i}\frac{\partial N_a^{\mathsf{neo}}}{\partial \xi_i^{\mathsf{M}}} + N_a^{\mathsf{neo}}\kappa^\rho$$
$$- g^{11}e_a n_a\frac{\partial \phi}{\partial \rho} + e_a n_a \hat{E}^\rho + \sum_{i=1}^{3} C_a^{\mathsf{Lor},i} n_a u_a^{\xi_i^{\mathsf{N}}} + R_a^\rho + m_a S_a u_{u_a}^\rho$$

where $C_a^{\mathsf{Lor},i}$ and R_a^ρ are the coefficient of the Lorentz force term and the contravariant radial friction force

$$C_a^{\mathsf{Lor},1} = 0, \quad C_a^{\mathsf{Lor},2} = \frac{e_a BI}{\psi'}, \quad C_a^{\mathsf{Lor},3} = -\frac{e_a B^2 R^2}{\psi'}$$
$$R_a^\rho \equiv \mp\frac{m_\mathrm{e} n_\mathrm{e}}{\tau_\mathrm{e}}u^\rho \mp \frac{3}{2\tau_\mathrm{e}\Omega_\mathrm{e}}\frac{I}{\sqrt{g}B}n_\mathrm{e}\frac{\partial T_\mathrm{e}}{\partial \chi}$$

- Equation for momentum in parallel direction

$$\frac{\partial}{\partial t}\left(m_a n_a u_{a\|}\right) = -\sum_{i=1}^{3} C_a^{\mathsf{kin},i} F_a^{\mathsf{kin},i} - \frac{\psi'}{\sqrt{g}B}\frac{\partial p_a}{\partial \chi} - \frac{\psi'}{\sqrt{g}B} N_a^{\mathsf{neo}} \frac{\partial \ln B}{\partial \chi} + \frac{2}{3}\frac{\psi'}{\sqrt{g}B}\frac{\partial N_a^{\mathsf{neo}}}{\partial \chi} + e_a n_a \hat{E}_\| + R_{a\|} + m_a S_{u_a} u_{a\|}$$

where $C_a^{\mathsf{kin},i}$ and $R_{a\|}$ are the coefficient of kinetic stress force in each direction and the parallel friction force

$$C_a^{\mathsf{kin},1} = \frac{\psi' g_{21}}{\sqrt{g}B}, \quad C_a^{\mathsf{kin},2} = \frac{\psi' g_{22}}{\sqrt{g}B}, \quad C_a^{\mathsf{kin},3} = \frac{I}{B}$$

$$R_{a\|} \equiv \mp \left\{ 0.51 \frac{m_\mathrm{e} n_\mathrm{e}}{\tau_\mathrm{e}}\left(u_{\mathrm{e}\|} - u_{\mathrm{i}\|}\right) + 0.71 \frac{\psi'}{\sqrt{g}B} n_\mathrm{e} \frac{\partial T_\mathrm{e}}{\partial \chi} \right\}$$

- Equation for momentum in toroidal direction

$$\frac{\partial}{\partial t}\left(m_a n_a u_a^\zeta\right) = - F_a^{\mathsf{kin},3} - \frac{I\psi'}{\sqrt{g}B^2R^2}N_a^{\mathsf{neo}}\frac{\partial \ln B}{\partial \chi} + \frac{I\psi'}{\sqrt{g}B^2R^2}\frac{\partial N_a^{\mathsf{neo}}}{\partial \chi} + N_a^{\mathsf{neo}}\kappa^\zeta + e_a n_a \hat{E}^\zeta + \frac{e_a\psi'}{R^2}n_a u_a^\rho + R_a^\zeta + m_a S_{u_a} u_a^\zeta$$

where R_a^ζ is the contravariant toroidal friction force

$$R_a^\zeta \equiv \mp \frac{m_\mathrm{e} n_\mathrm{e}}{\tau_\mathrm{e}}\left\{\left(u_\mathrm{e}^\zeta - u_\mathrm{i}^\zeta\right) - 0.49\frac{I}{BR^2}\left(u_{\mathrm{e}\parallel} - u_{\mathrm{i}\parallel}\right)\right\} \mp 0.71\frac{\psi' I}{\sqrt{g}B^2R^2}n_\mathrm{e}\frac{\partial T_\mathrm{e}}{\partial \chi} \mp \sum_{i=1}^{3}\frac{3}{2\tau_\mathrm{e}\Omega_\mathrm{e}}\frac{\psi' g^{1i}}{BR^2}n_\mathrm{e}\frac{\partial T_\mathrm{e}}{\partial \xi_i^{\mathsf{M}}}$$

Equation for internal energy is obtained by transforming equation for energy transport of Braginskii's equation into the advection-diffusion form

- Equation for internal energy

$$\frac{\partial}{\partial t}\left(\frac{3}{2}p_a\right)+\nabla\cdot\left(p_a\boldsymbol{u}_{p_a}-n_a\overset{\leftrightarrow}{\chi}_a\cdot\nabla T_a\right)=Q_{p_a}$$

where $\boldsymbol{u}_{p_a}\equiv(5/2)\boldsymbol{u}_a+p_a^{-1}\boldsymbol{q}_{u_a}$, Q_{p_a} and $\overset{\leftrightarrow}{\chi}_a$ are the the energy flow velocity, the energy source term and the diffusion coefficient tensor and respectively.

Current program design

Variables for TASK/T2:

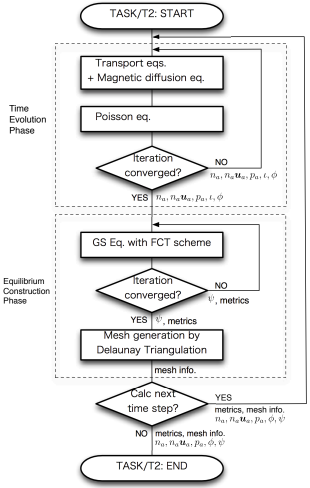
$n_a,\ p_a,\ n_a u_a^{\rho},\ n_a u_{a\parallel},\ n_a u_s^{\zeta}$
$\phi(\rho),\ \iota(\rho),\ \psi(R,Z)$

Metrics

$g_{ij},\ g^{ij},\ \sqrt{g}$

Mesh info.

Boundary Condition,
Element-Node Relation,
Node-Node Connectivity, etc...



Brief flowchart of TASK/T2
(under development)

Current status and issues of TASK/T2: TE-phase

Employed algorithms

- Discretization:
  - SUPG-FEM: Transport eqs.
  - BG-FEM: Magnetic diffusion eq. and Poisson eq.
- Element: Multi-scale rectangular element
- Matrix Solver: Krylov subspace iterative method (PETSc library)
- Nonlinear Solver: Picard iteration

We have derived transport eqs. and been coding TE-Phase; however, there are still remains some issues in transport modeling.

- Appropriate modeling of gyro-viscous force in core region
- Appropriate energy cancellation between diamagnetic terms

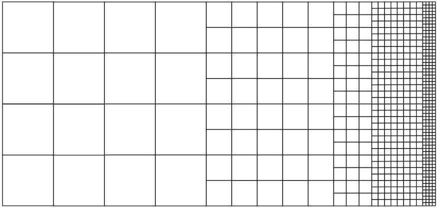
Current status and issues of TASK/T2: EC-phase

Grad-Shafranov Eq. with FCT scheme

- TASK/EQU
  - Free boundary 2D MHD equilibrium solver with FCT scheme included in integrated toroidal plasma modeling code TASK.

Mesh generation

- Multi-scale structural rectangular mesh generation algorithm



Basic concept of multi-scale structual rectangular mesh

Summary and Future works

Summary

- A set of equations required for two-dimensional transport modeling for tokamak plasmas has been derived for integrated analysis of core and peripheral plasmas
  - Transport equations are derived from Braginskii's equations with the neoclassical viscosity in MFCS and reduced to two-dimensional with toroidal axisymmetry.
  - By combining these transport equations with the electromagnetic equations, a more self-consistent two-dimensional transport analysis including the field evolution will be available.

Future works

- Developing the two-dimensional transport code using the FEM to simulate time evolution of tokamak plasmas.
  - Analysis of the asymmetric effect in limiter configuration
  - Full 2D transport analysis in entire tokamak plasma