Development of RWM analysis code for rotating plasmas
(RWM = Resistive Wall Mode)

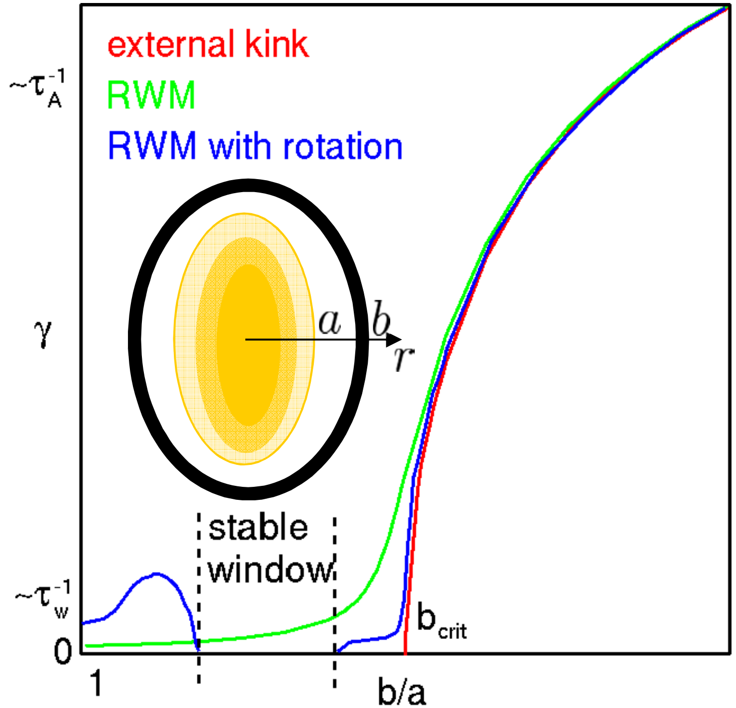
J. Shiraishi, N. Aiba, and M. Yagi (JAEA)

Acknowledgements : M.S. Chu and L.L. Lao (GA)
T. Ozeki (JAEA)

Why RWM? What's RWM? How to stabilize RWM?

Stabilization of RWM is inevitable for advanced tokamaks aiming at steady state high-β operation such as JT-60SA.

- Originates from low-n external kink modes (timescale ~ τ_A)
- Ideal wall → stabilization of ideal external kink
- Resistive wall → slow down kink instability to timescale of eddy current decay time in the resistive wall → RWM (timescale ~ τ_w)
- Many theoretical/experimental research on rotational stabilization of RWM

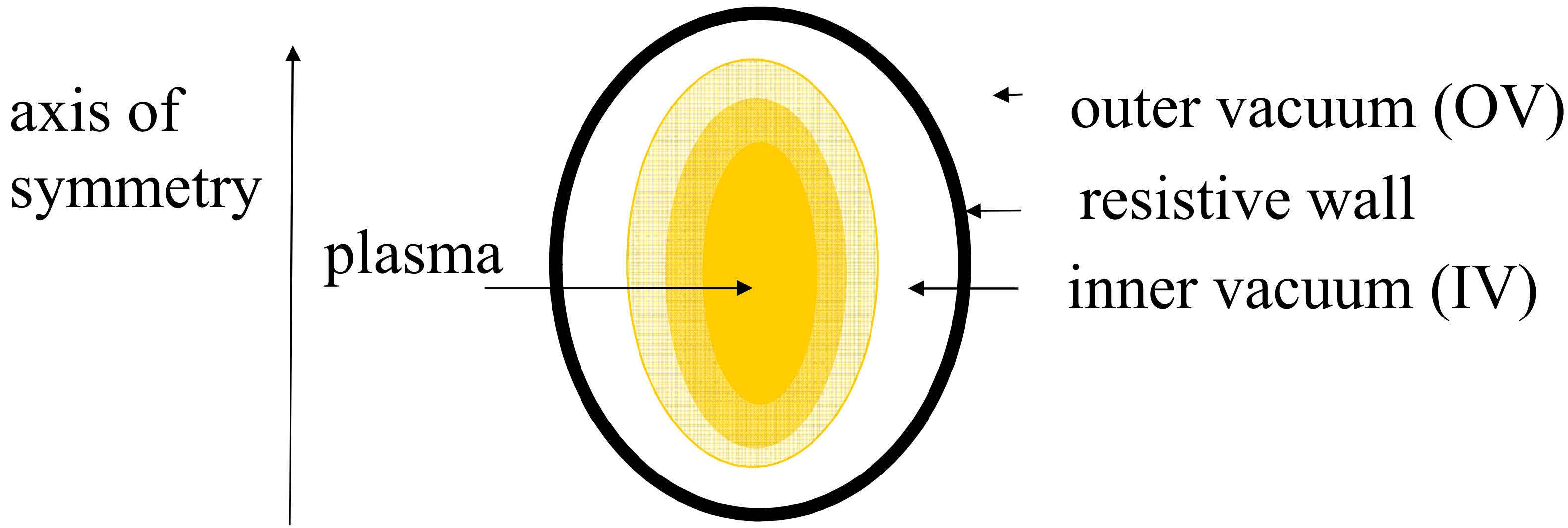


As a basis of quantitative RWM study, we need to develop a numerical code for RWM in realistic tokamak geometry including plasma rotational effects.

Introduction – RWM codes in tokamak geometry

- MARS-F (Chu PoP05), MARS-K (Liu NF09), CarMa (Liu PoP09)
  - Linearized resistive MHD, perturbative toroidal rotation, kinetic effects, 3D wall, feedback.
- NMA (Chu NF03)
  - Linearized ideal MHD, feedback.
- MISK (Berkery PRL11)
  - Linearized ideal MHD, without rotation, kinetic effects
- VALEN (Bialek PoP01)
  - Linearized ideal MHD, without rotation, 3D wall, and feedback
- **RWMaC/MINERVA**
  - **We develop a new RWM code. It has some advantages : (1) perturbative poloidal rotation (Aiba PoP11) (2) equilibrium change by toroidal rotation (3) initial value approach**

Energy balance in plasma – wall – vacuum system (Shiraishi PoP10)

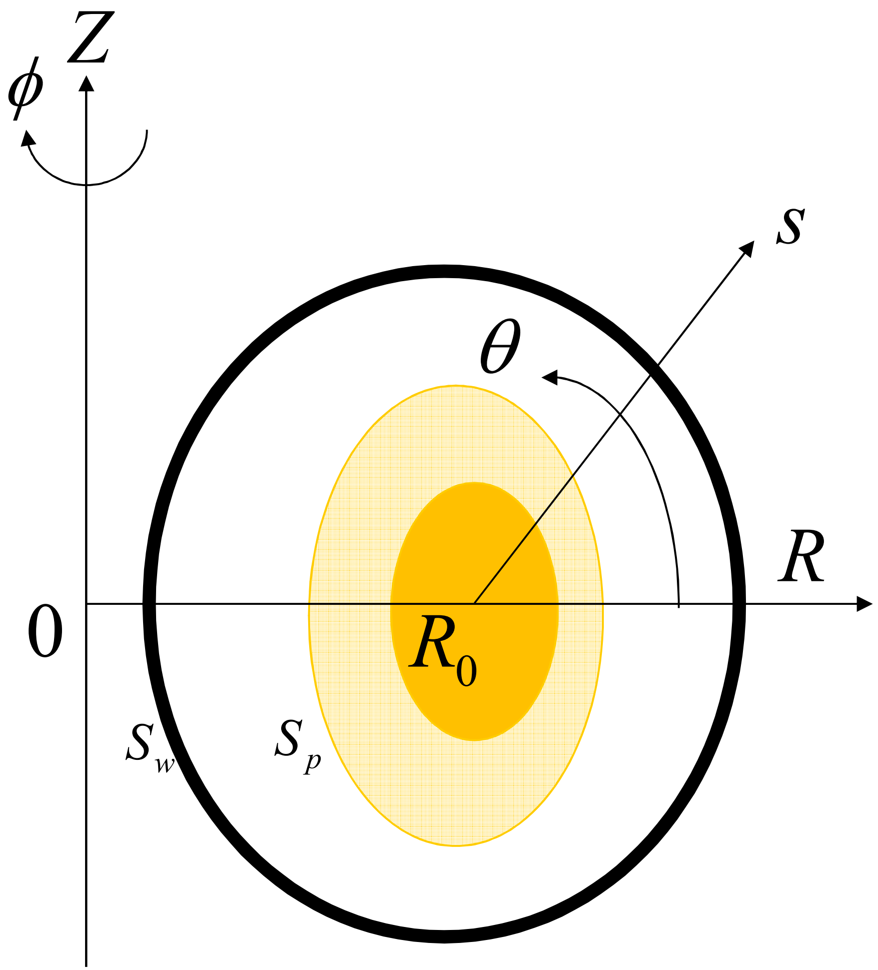


$$W_{RWM} = K + U + W_p - W_d + W_{OV} + W_{IV} + D_W = 0$$

plasma: $K + U + W_p - W_d$; vacuum: $W_{OV} + W_{IV}$; wall: D_W

- K: kinetic energy
- U: convective term
- $W_p - W_d$: potential energy with rotation
- W_{OV}: vacuum magnetic energy in OV
- W_{IV}: vacuum magnetic energy in IV
- D_W: energy dissipation in resistive wall

RWMaC geometry



- We compute all metric quantities in IV in a new coordinate system (s, θ, ϕ).
- We use a "thin-shell approximation," which indicates that normal magnetic field is continuous across the wall.

Governing equations in resistive wall

Governing equations are the pre-Maxwell equations and the Ohm's law.

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \qquad \partial_t \mathbf{B} = -\eta \nabla \times \mathbf{J}$$

η :volume resistivity of wall

We introduce a "current potential" and magnetic "scalar" potentials in vacuum:

$$\mathbf{J}(s,\theta,\phi,t) = \nabla s \times \nabla \kappa(\theta,\phi,t)\delta(s - s_w)$$

$$\mathbf{B} = \nabla \chi^{(\pm)} \quad \text{in OV (IV)}$$

Integration of the Ampere's law on the wall yields a "jump" condition for magnetic field.

$$\chi^{(w+)}(\theta,\phi,t) - \chi^{(w-)}(\theta,\phi,t) = \mu_0 \kappa(\theta,\phi,t) \qquad (1)$$

The superscript (w) indicates the limit to the resistive wall.
Integration of the Faraday's law gives a "diffusion equation" for magnetic energy.

$$\frac{\Delta |\nabla s|}{\eta} \frac{\partial B^{(n)}}{\partial t} = L\kappa \qquad (2)$$

Δ :wall width measured by s

$B^{(n)} = \mathbf{B} \cdot \hat{n}$:normal ***B*** on wall

L :elliptic operator (shown in the next page)

Method of eigenfunction expansion

We invoke a property of operator L by considering following eigenvalue problem on wall:

$$L\kappa = \omega |\nabla s| \kappa \quad (3)$$

$$L\bullet = -\nabla s \times [\nabla \times (\nabla s \times \nabla \bullet)]$$

We can easily prove that

(a) L is positive, i.e., $^\forall \omega > 0, ^\forall \omega \in \mathbb{R}$

(b) Eigenfunctions belonging to different eigenvalues are orthogonal

$$\int_{S_w} \kappa_j^* \kappa_k |\nabla s| \sqrt{g} d\theta d\phi = 0$$

By (2), we can expand $B^{(n)}$ and κ on wall as

$$B^{(n)}(\theta,\phi,t) = \sum_{j=1}^{N_p} a_j(t) \hat{\kappa}_j(\theta,\phi) \quad (4)$$

$$\kappa(\theta,\phi,t) = \frac{R_0^3 \Delta}{\eta} \sum_{j=1}^{N_p} \frac{1}{\omega_j} \frac{da_j}{dt} \hat{\kappa}_j(\theta,\phi) \quad (5)$$

$\hat{\kappa}_j$:normalized eigenfunction

N_p :poloidal mode number

Energy balance

To get energy balance of the system, we multiply (1) by $(1/2\mu_0)B^{(n)*}|\nabla s|\sqrt{g}d\theta d\phi$ and integrate them on wall.

$$\frac{1}{2\mu_0}\int_{S_w}\chi^{(w+)}B^{(n)*}|\nabla s|\sqrt{g}d\theta d\phi-\frac{1}{2\mu_0}\int_{S_w}\chi^{(w-)}B^{(n)*}|\nabla s|\sqrt{g}d\theta d\phi=\frac{1}{2}\int_{S_w}\kappa B^{(n)*}|\nabla s|\sqrt{g}d\theta d\phi$$

The RHS gives energy dissipation in the resistive wall, which by (4) and (5) can be written as

$$\frac{1}{2}\int_{S_w}\kappa B^{(n)*}|\nabla s|\sqrt{g}d\theta d\phi=\frac{R_0^5\Delta}{\eta}\sum_{j=1}^{N_p}\frac{1}{\omega_j}a_j^*\frac{da_j}{dt}=D_W \qquad (6)$$

After some manipulation, we get energy balance:

Plasma response

$$W_{RWM}=W_{OV}+W_{IV}+D_W+\frac{1}{2\mu_0}\int_{S_p}\chi^{(p+)}\mathbf{Q}_e^*\cdot\hat{n}dS=0 \quad (7)$$

$$W_{IV(OV)}=\frac{1}{2\mu_0}\int_{IV(OV)}|\nabla\chi|^2\,dV$$:vacuum magnetic energy

Plasma response

We employ Frieman-Rotenberg equation (Frieman, RMP 60) as a plasma model, linearized ideal MHD equation including equilibrium rotation.

$$\rho_0 \frac{\partial^2 \boldsymbol{\xi}}{\partial t^2} + 2\rho_0 (\mathbf{V}_0 \cdot \nabla) \frac{\partial \boldsymbol{\xi}}{\partial t} = F\boldsymbol{\xi}$$

F :generalized force operator

Energy balance in plasma leads to

$$\frac{1}{2\mu_0}\int_{S_p} \chi^{(p+)} \mathbf{Q}_e^* \cdot \hat{n} dS = K + U + W_p - W_d + \frac{1}{2}\int_{S_p} \left[\left(\frac{\mathbf{Q}\cdot\mathbf{B}}{\mu_0} + p \right) \boldsymbol{\xi}^* - \left(\frac{\mathbf{Q}_e \cdot \mathbf{B}_e}{\mu_0} \right) \boldsymbol{\xi}^* \right] \cdot \hat{n} dS$$

natural BC

$$K = \frac{1}{2}\int \boldsymbol{\xi}^* \cdot \rho_0 \frac{\partial^2 \boldsymbol{\xi}}{\partial t^2} dV$$: kinetic energy

$$U = \frac{1}{2}\int \boldsymbol{\xi}^* \cdot 2\rho_0 (\mathbf{V}_0 \cdot \nabla) \frac{\partial \boldsymbol{\xi}}{\partial t} dV$$: energy associated with convective term

W_p, W_d : potential energy

Thus (7) yields energy balance for the resistive wall-plasma system:

$$W_{RWM} = K + U + W_p - W_d + W_{OV} + W_{IV} + D_W = 0 \quad (8)$$

`MINERVA` (Aiba, CPC09) calculates these terms by FEM and Fourier decomposition.

“RWMaC” modules compute $W_{IV(OV)}$ and D_W

Inner vacuum and outer vacuum : $W_{IV(OV)}$

Governing equation : Laplace equation for χ
(magnetic scalar potential $B = \nabla\chi$)

Numerical scheme : FEM for IV
FEM or Green’s function method for OV

Resistive wall : D_W

Governing equation : diffusion equation for κ
current potential $J = (\nabla s \times \nabla\kappa)\delta(s - s_{wall})$

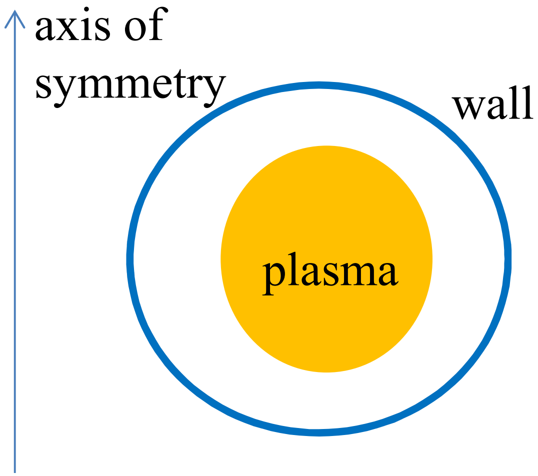
Numerical scheme : FEM

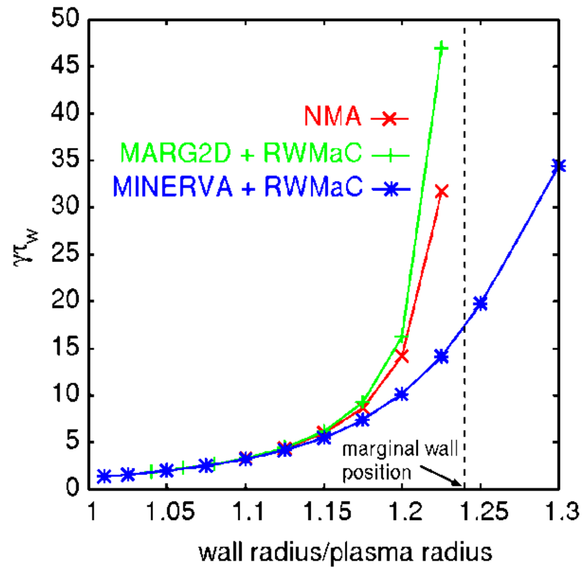
Boundary conditions on resistive wall and plasma surface :

Continuity of normal magnetic field + natural boundary condition

Implementation of RWMaC in MINERVA

The RWMaC module solves electromagnetic problems in the vacuum and wall. The MINERVA [Aiba CPC (2009)] solves linear plasma dynamics with equilibrium toroidal rotation.



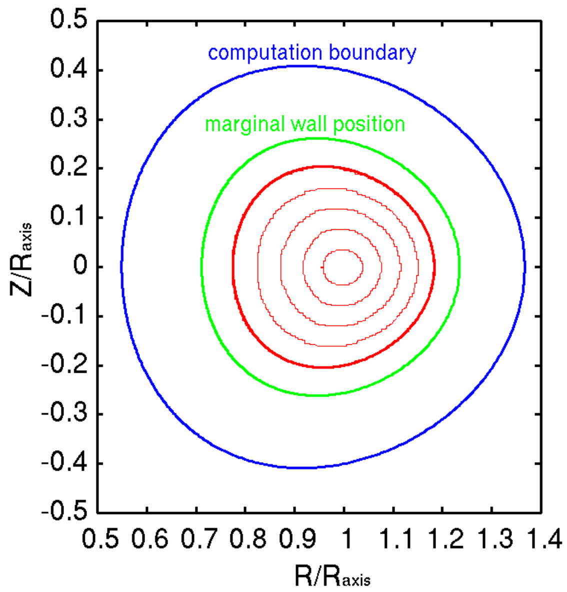


MINERVA/RWMaC has been benchmarked with NMA (Chu NF03) using Solov'ev equilibrium.

MINERVA/RWMaC has some advantages (1) include poloidal rotation effect perturbatively (Aiba PoP11) (2) include MHD equilibrium change induced by toroidal rotation. (3) can solve initial value problem.

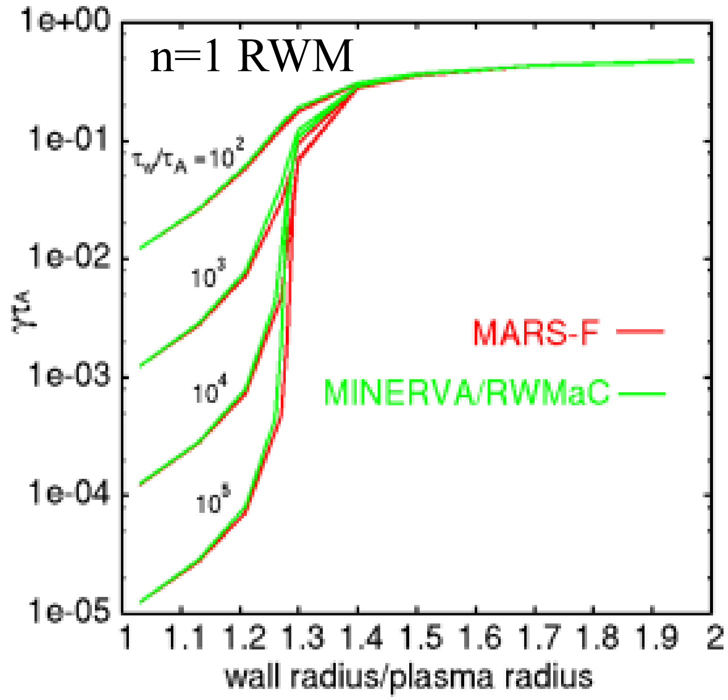
Benchmark between MINERVA/RWMaC and MARS-F using Solov'ev circle without rotation

We start benchmarking from Solov'ev circle without rotation to remove numerical errors in numerical computation of the Grad-Shafranov equation.



$\beta_N = 3.3, B_{axis} = 1\mathrm{T}, I_p = 0.17\mathrm{MA},$

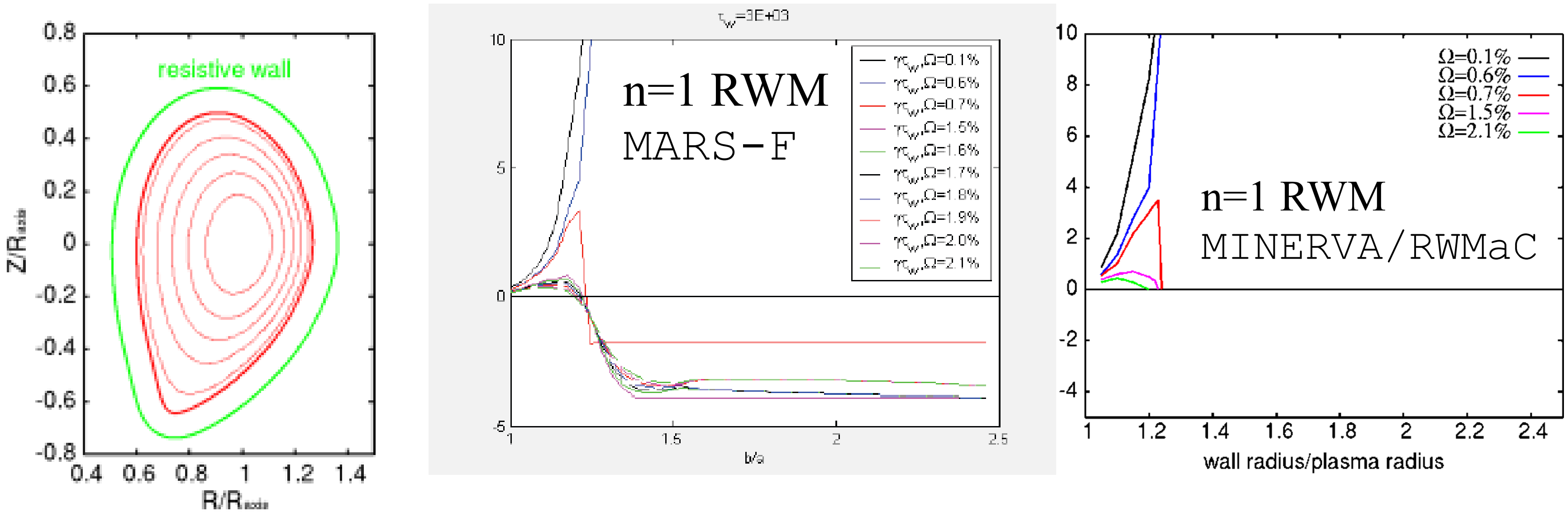
$q_0 = 1.2, q_a = 1.4$



Benchmark succeeded for the marginal wall position of external kink, and the RWM growth rates even for large wall decay time.

Benchmark of MINERVA/RWMaC with MARS-F using up-down asymmetric MHD equilibrium with toroidal rotation

MINERVA/RWMaC has been benchmarked with MARS-F (Chu PoP95) with rigid toroidal rotation under assumption that toroidal rotation does not affect MHD equilibrium.



$\beta_N = 2.63, B_{axis} = 1.7\text{T}$

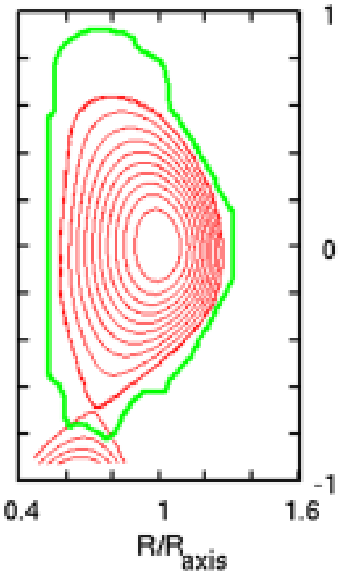
$I_p = 1.1\text{MA}$

Benchmark succeeded for frequency of $O(1\%)$ of Alfven frequency in RWM growth rate, critical rotation, location of stable window.

(Many thanks to L.L. Lao and M.S. Chu)

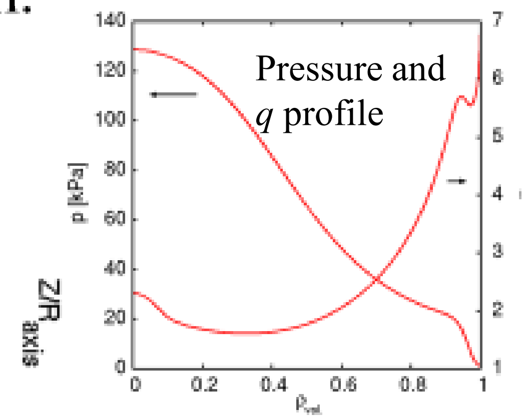
Application to RWM analysis in JT-60SA

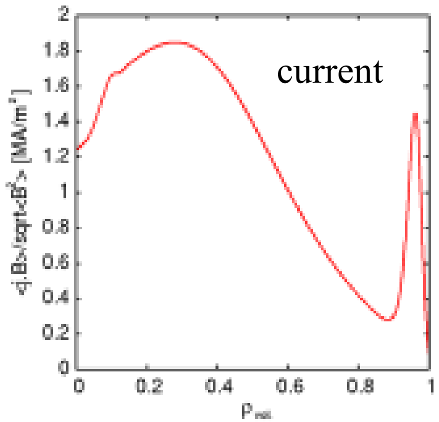
By modeling the wall shape, we start RWM study in JT-60SA configuration.



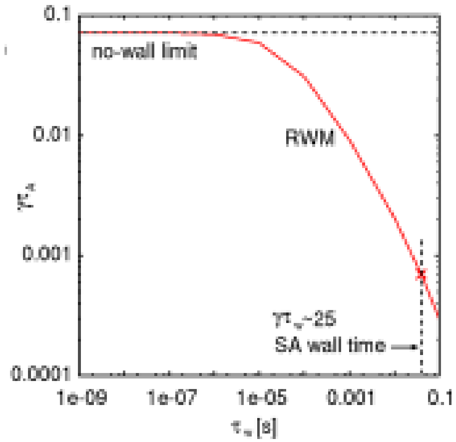
$\beta_N = 3.3, B_{axis} = 1.7\text{T}$

$I_p = 2.3\text{MA}$



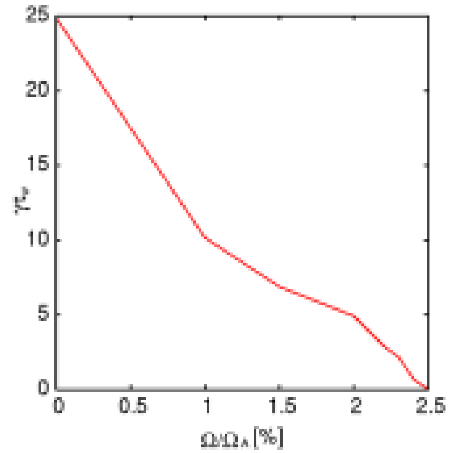


n=1 RWM growth rate vs. wall time



Estimated RWM growth rate $\gamma \sim 0.6$kHz for stabilizing plate

RWM growth rates vs. rigid rotation frequency



Rigid rotation requires $\Omega \sim 0.025\omega_A$ to fully stabilize RWM.

Summary

- We have
  - developed `RWMaC` modules that solve electromagnetic problems in the vacuum and resistive wall.
  - implemented `RWMaC` modules in linear MHD code, `MINERVA` (with rotation) and `MARG2D` (w/o rot, inertia).
  - benchmarked `MINERVA/RWMaC` against `NMA` (w/o rotation) and `MARS-F` (with rotation).
  - used `MINERVA/RWMaC` to study RWMs in JT-60SA high β_N equilibrium.
- Based on `MINERVA/RWMaC`, we will
  - study how the equilibrium change induced by toroidal rotation affects RWMs.
  - implement kinetic effects.
  - implement the 3D wall structure.