

# **Magnetic island evolution in hot ion plasmas**

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# Outline

- 1. Introduction**
- 2. Forced island propagation**
  - 1. Islands in helical plasmas**
- 3. Natural island propagation**
  - 1. Islands in tokamak plasmas**
- 4. Summary**

# Neoclassical tearing mode

- NTM makes magnetic islands and limits achievable beta in high performance tokamaks and ITER.

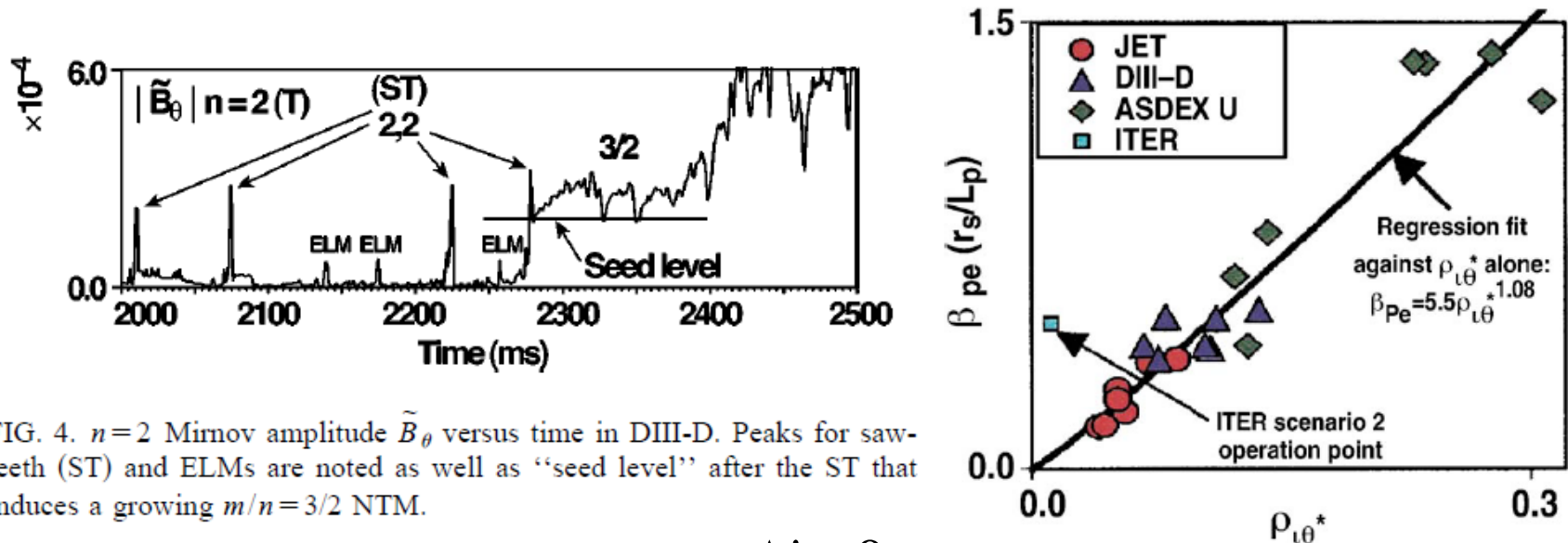


FIG. 4.  $n=2$  Mirnov amplitude  $\tilde{B}_\theta$  versus time in DIII-D. Peaks for sawteeth (ST) and ELMs are noted as well as “seed level” after the ST that induces a growing  $m/n=3/2$  NTM.

R. J. La Haye, et al., Phys. Plasmas (2000)  $\Delta' < 0$

R. J. La Haye., Phys. Plasmas (2006)

- The polarization current effect is responsible for understanding the  $\rho_{10}^*$  scaling of the NTM excitation.
- Ion temperature is high in core region.
- The first numerical simulation of the polarization current in hot ion plasmas is presented.

# Island equation for NTM excitation

- Evolution of magnetic island width

(Rutherford equation)

$$\frac{dW}{dt} = \Delta' + \Delta_{bs} + \Delta_{pol} \quad \Delta' < 0 \quad W \text{ Island width}$$

**Bootstrap  
current term**

$$\Delta_{bs} = C_1 \frac{W}{W^2 + W_c^2}$$

$\omega$  Island frequency

**Polarization  
term**

$$\Delta_{pol} = - \int_0^{L_x} \frac{dx}{L_x} \int_0^{L_y} \frac{dy}{L_y} \frac{1}{\beta\Psi} J_{pol} \cos k_{1y} y = C_2 \frac{\omega(\omega - \omega_{*i})}{W^3}$$

Connor et al. Phys. Plasmas (2001)

# Rutherford equation including polarization current

• 1

$$\frac{\partial}{\partial t} \nabla_{\perp}^2 \Phi + [\Phi, \nabla_{\perp}^2 \Phi] = -\nabla_{\parallel} J$$

$$\frac{\partial \psi}{\partial t} = -\nabla_{\parallel} \Phi + \nabla_{\parallel} n + \eta J$$

$$\frac{\partial}{\partial t} n + [\Phi, n] = -\nabla_{\parallel} J$$

$$[\Phi, \nabla_{\perp}^2 \Phi] = -\nabla_{\parallel} J$$

$$\frac{\partial \psi}{\partial t} = \eta J - \nabla_{\parallel} (\Phi - n)$$

$$[\Phi, n] = -\nabla_{\parallel} J$$

$$\psi = \frac{x^2}{2L_s} + \Psi(t) \cos ky$$

$$\langle B \nabla_{\parallel} f \rangle_{\psi} = 0$$

$$B = 1$$

$$T_e = 1$$

• 2 Rutherford equation

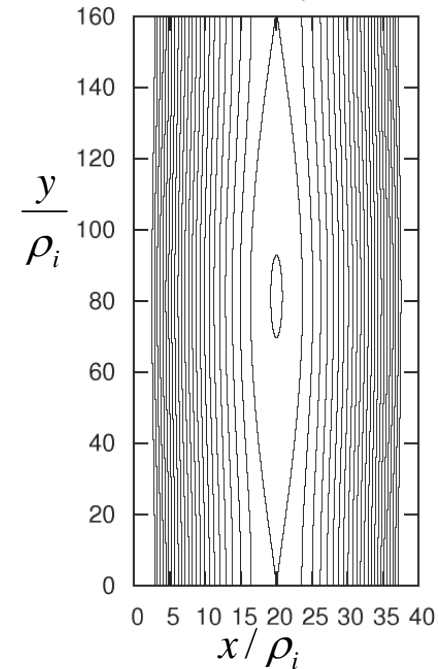
$$J = \langle J \rangle + J_{pol}, \quad \left\langle \frac{\partial \psi}{\partial t} \right\rangle = \eta \langle J \rangle = \eta (J - J_{pol})$$

$$\int \left\langle \frac{\partial \psi}{\partial t} \right\rangle \cos(ky) dx = \int \eta (J - J_{pol}) \cos(ky) dx$$

$$\frac{C}{\eta} \frac{dW}{dt} = \Delta' + \Delta_{pol}$$

$$W = 4\sqrt{\Psi L_s}$$

$$C = 0.823$$



• 3 Polarization current

Connor et al. Phys. Plasmas (2001)

$$\Phi - n = H(\psi)$$

$$\nabla_{\parallel} J = -[\Phi, n] = \frac{dH}{d\psi} [\psi, \Phi]$$

$$J_{pol} = J - \langle J \rangle = \frac{dH}{d\psi} (\Phi - \langle \Phi \rangle)$$

$$[\Phi, \nabla_{\perp}^2 \Phi - n] = 0$$

$$\nabla_{\perp}^2 \Phi = I(\Phi) - H(\psi)$$

# 2D reduced two-fluid equations

$$\frac{d}{dt} n = -\nabla_{\parallel} v_{e\parallel} + \mu \nabla_{\perp}^2 n \quad \text{Electron density}$$

$$\frac{d}{dt} v_{\parallel} = -\beta \nabla_{\parallel} p + \mu \nabla_{\perp}^2 v_{\parallel} \quad \text{Parallel velocity}$$

$$\frac{d}{dt} \nabla_{\perp}^2 \Phi = -\nabla_{\parallel} J + \nabla_{\perp} \cdot [\nabla_{\perp} \Phi, p_i] / \rho_* + \nu \nabla_{\perp}^2 \nabla_{\perp}^2 (\Phi + p_i) \quad \text{Vorticity equation}$$

$$\eta J = \nabla_{\parallel} \Phi - \nabla_{\parallel} p_e \quad \text{Generalized Ohms law}$$

$$\frac{d}{dt} T_i = -(\Gamma - 1) T_{eq} \nabla_{\parallel} v_{\parallel} - (\Gamma - 1) \kappa_L T_i + \mu \nabla_{\perp}^2 T_i \quad \text{Ion temperature}$$

$$J = J_{pol}$$

$$p = p_i + p_e$$

$$p_i = T_i n$$

$$p_e = \tau_e n$$

$$v_{e\parallel} = v_{\parallel} + J$$

$$\rho_* = \rho_i / L$$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + [\Phi, f] / \rho_*$$

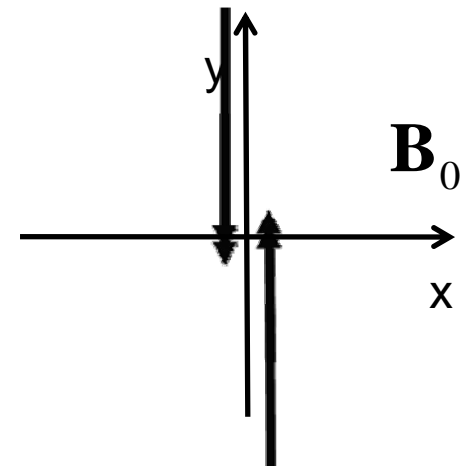
$$\mathbf{v}_E = \mathbf{b} \times \nabla \Phi$$

$$\nabla_{\parallel} f = -\beta [\psi, f] / \rho_*$$

$$\psi = \frac{x^2}{2L_s} + \Psi \cos ky$$

$$\beta = 0.01$$

$$\eta_i = 0$$



# Forced island propagation

- Force acting on magnetic island and polarization current in helical plasmas

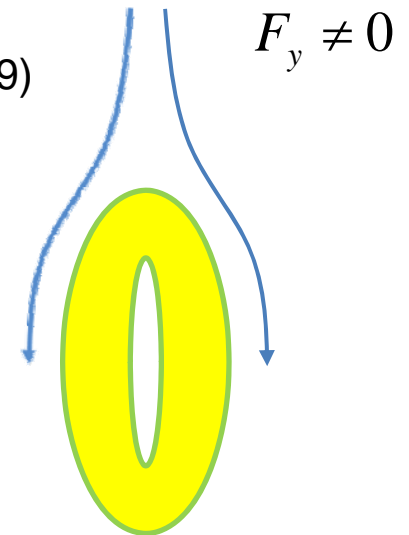
- Incompressible plasmas ( $V_{\parallel} = 0$ )

F. Waelbroeck, et.al, Plasma Phys. Controlled Fusion (2009)

- Cold ion plasmas ( $T_i = 0$ )

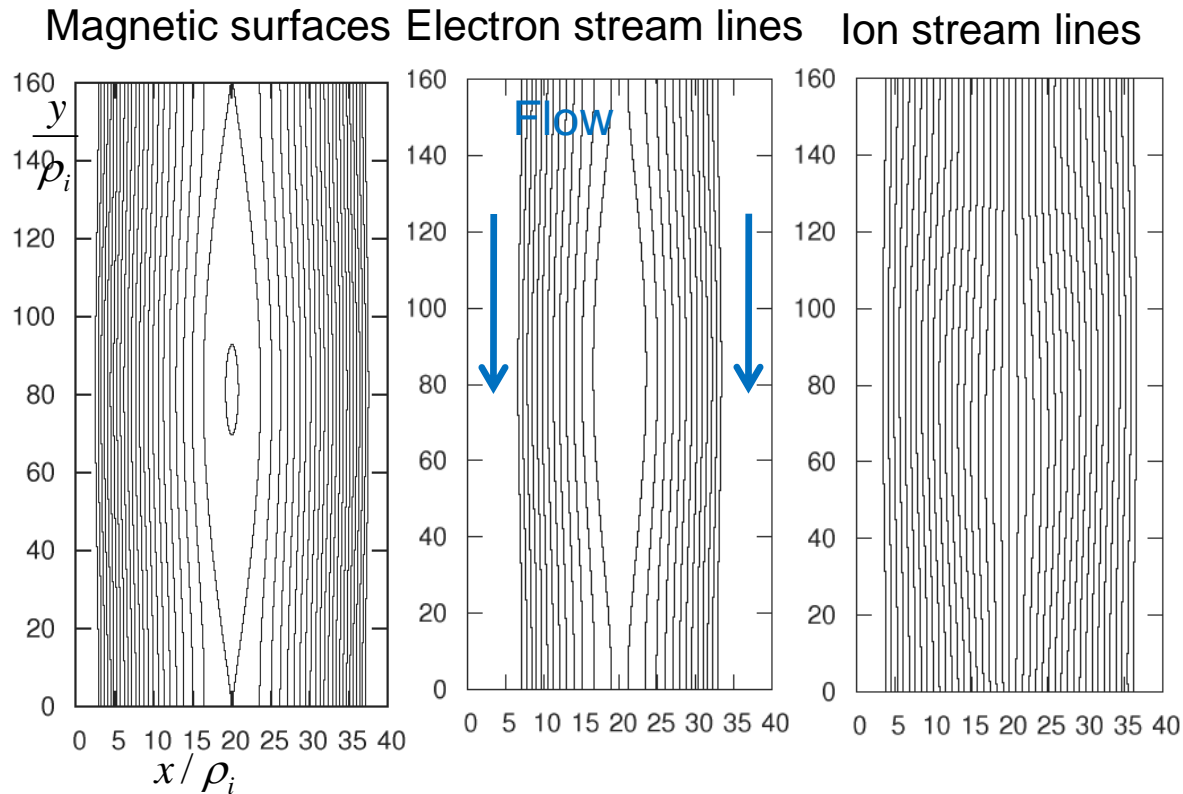
R. Fitzpatrick, et.al, Phys. Plasmas (2006)

- Hot ion plasmas



# Forced magnetic island propagation

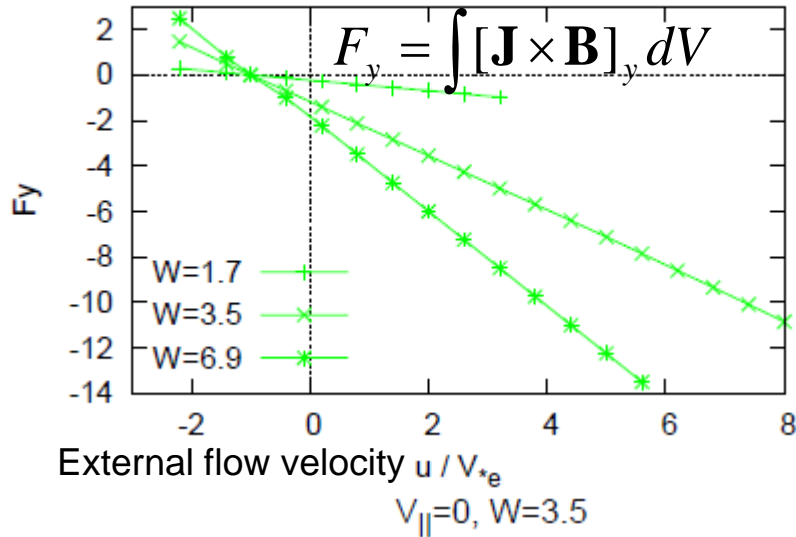
- Simulations are carried out in the island fixed frame.



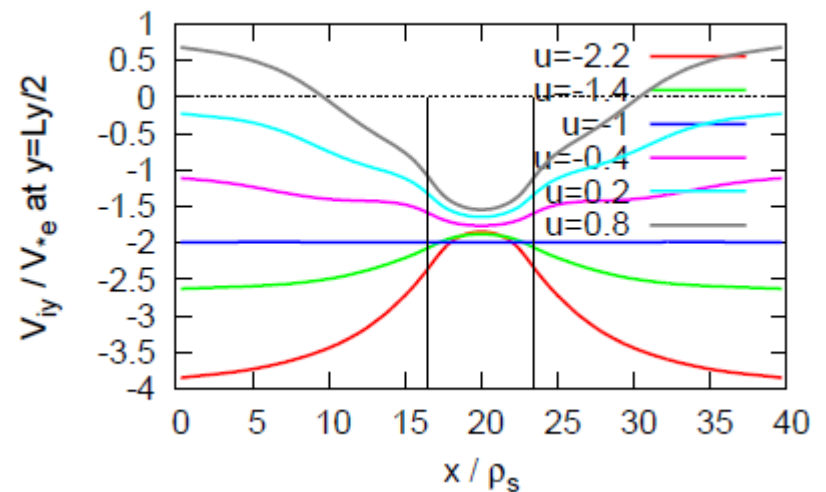
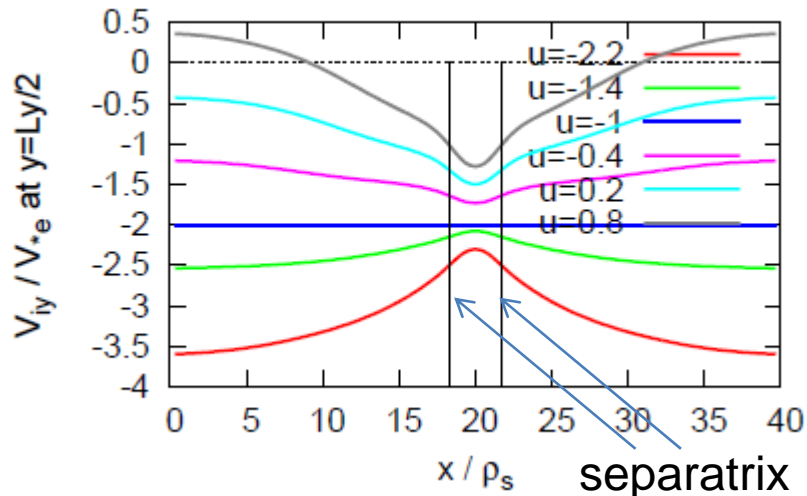
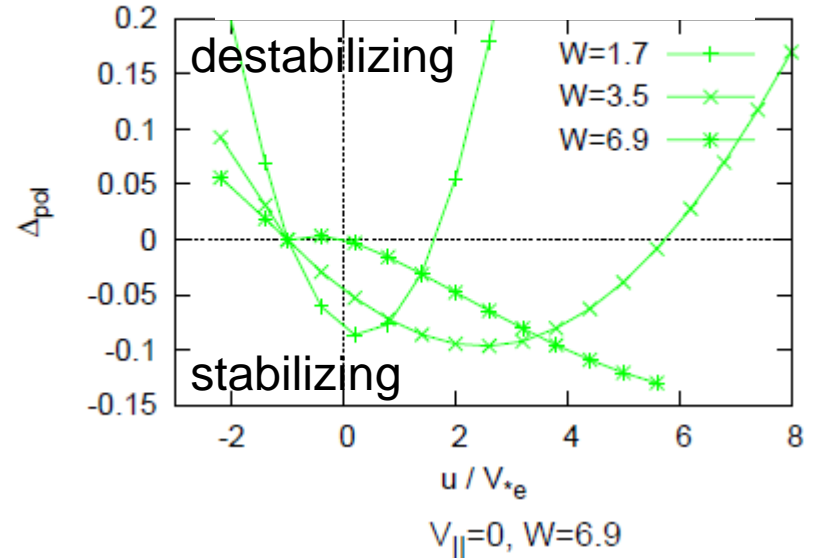


# Incompressible plasmas ( $V_{\parallel} = 0$ )

Electromagnetic force



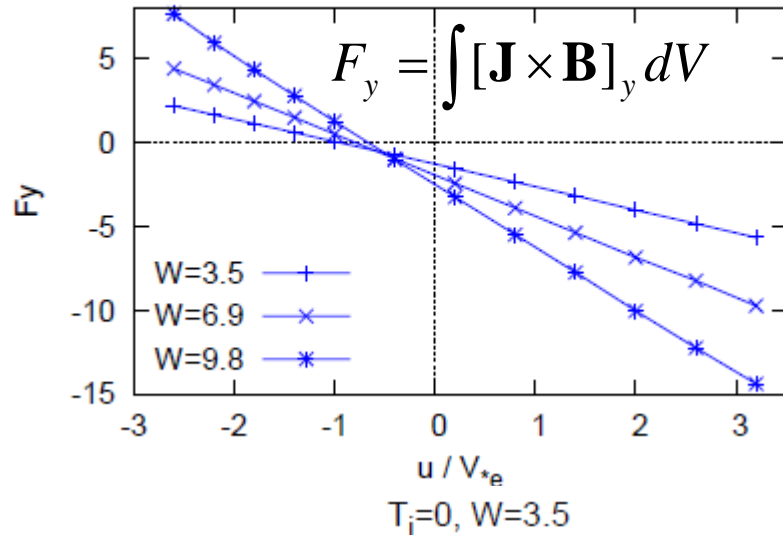
Polarization current



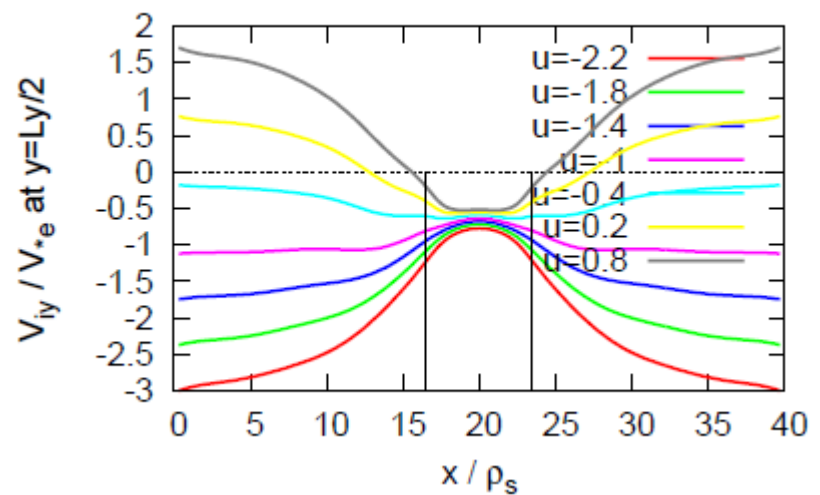
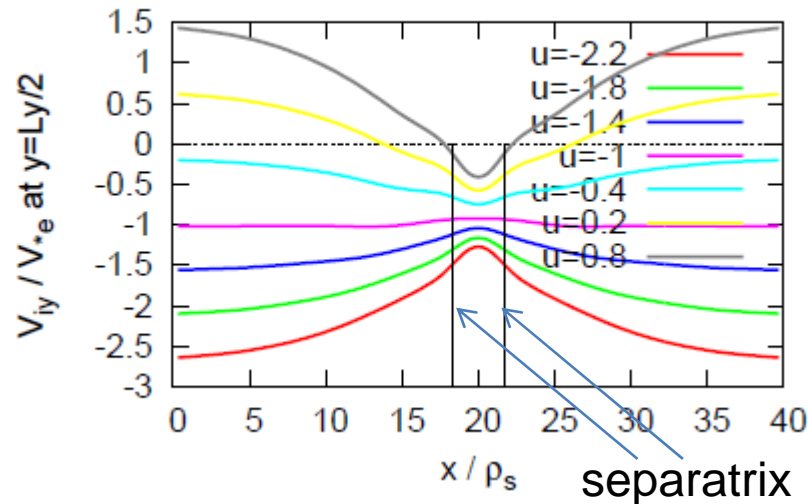
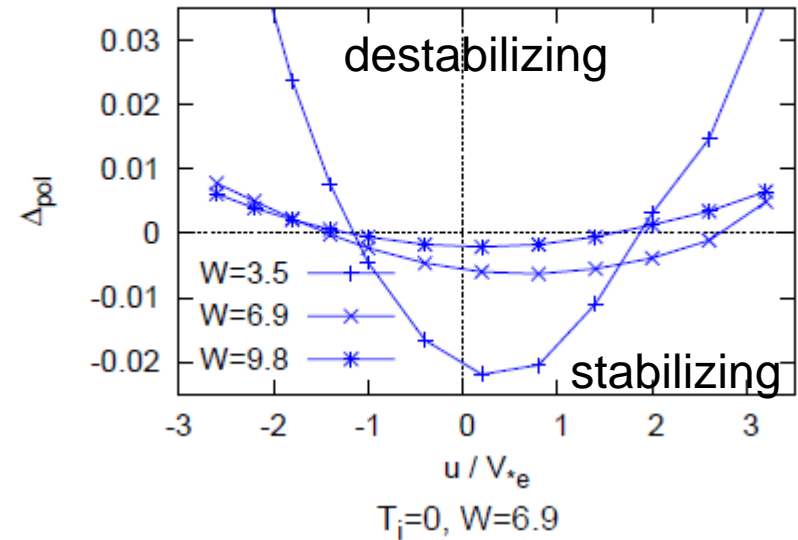
- When the applied ExB flow totally cancel out the equilibrium electron diamagnetic flow,  $u = -1$ ,  $F_y = 0$  and  $\Delta'_{pol} = 0$ .

# Cold ion plasmas (Ti=0)

Electromagnetic force

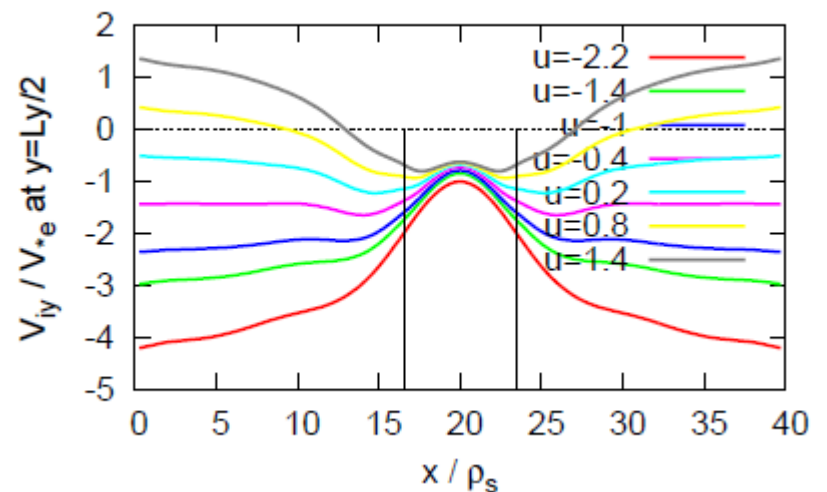
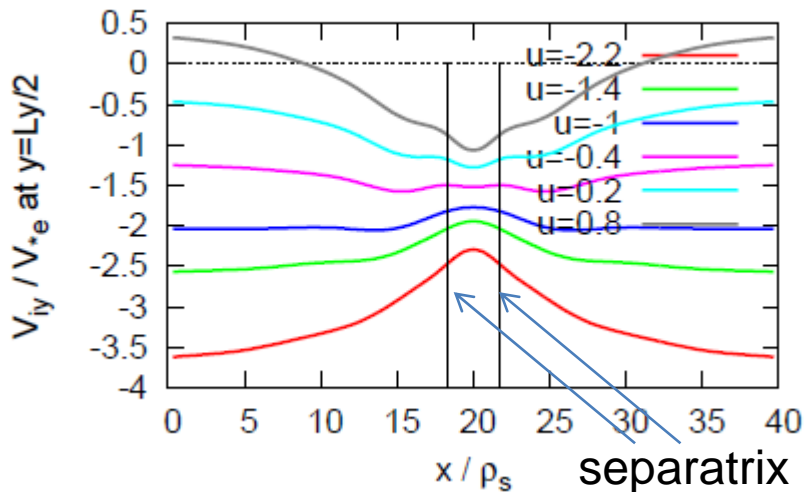
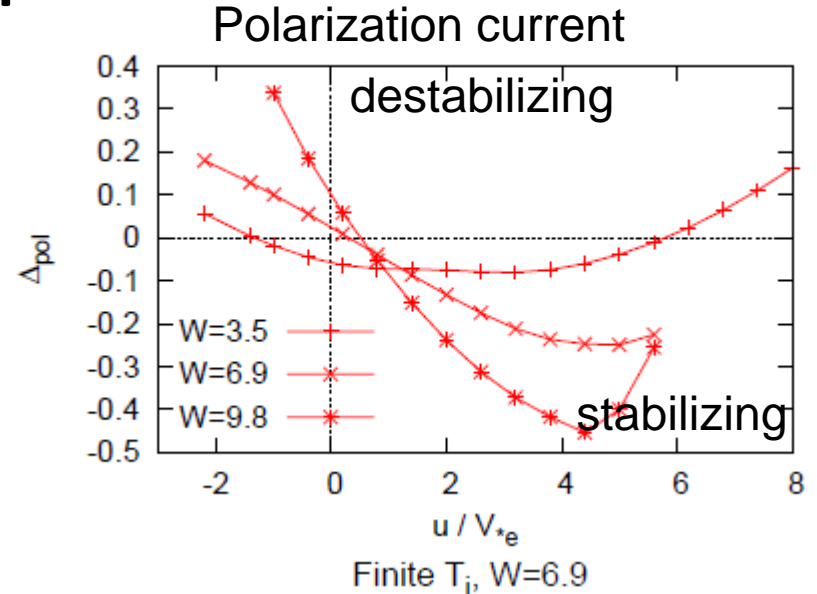
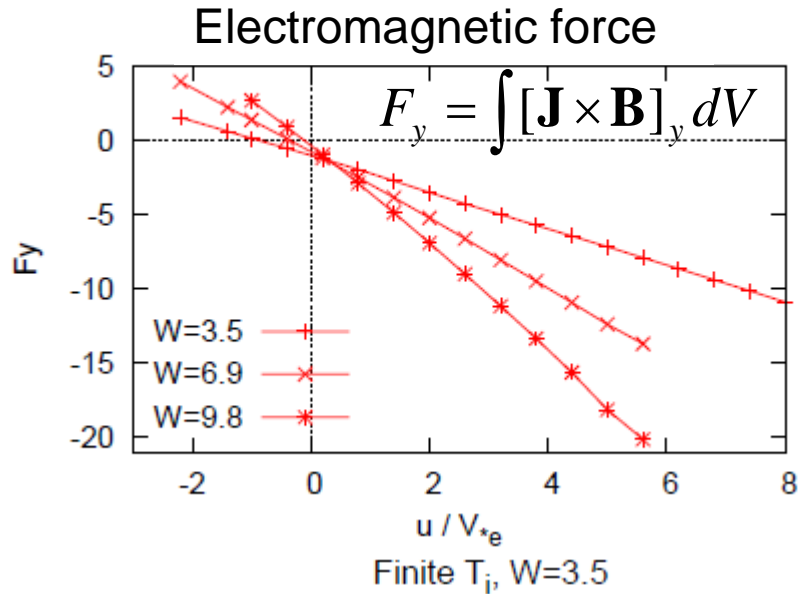


Polarization current



- In compressible plasmas, the sound wave propagation flattens density inside the island, so that the diamagnetic velocity is reduced. 10

# Hot ion plasmas

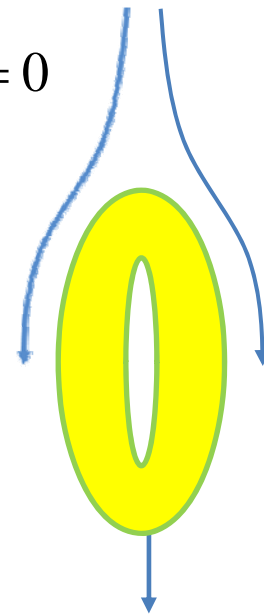


- The polarization current is an order of magnitude larger in hot than in cold ion plasmas.

# Natural magnetic island propagation

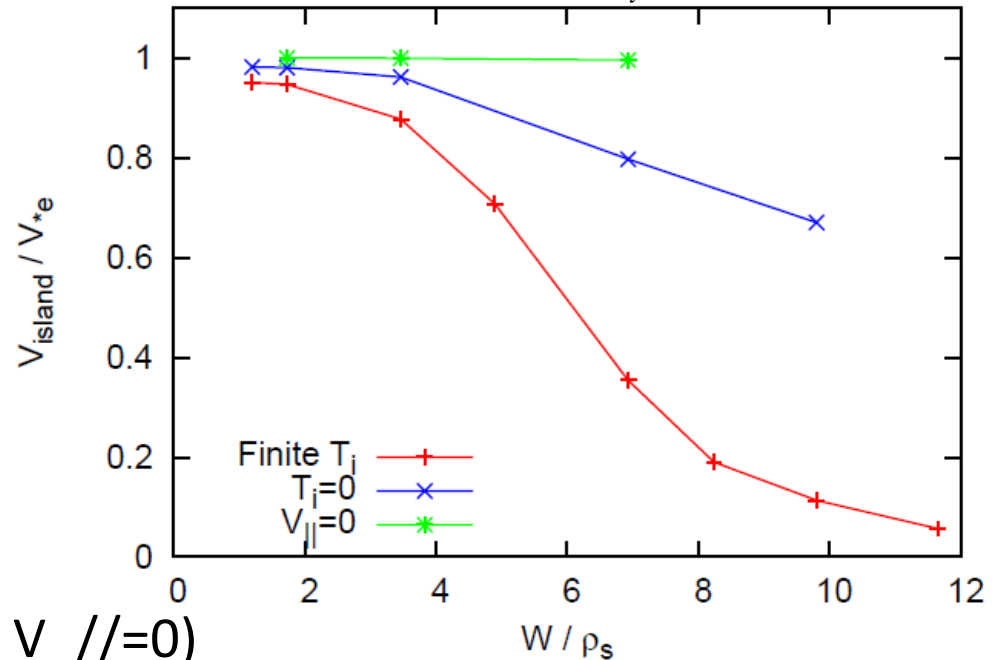
- Island propagation velocity and polarization current in tokamak plasmas

$$F_y = \int [\mathbf{J} \times \mathbf{B}]_y dV = 0$$



# Propagation velocity $F_y(V_{island}) = 0$

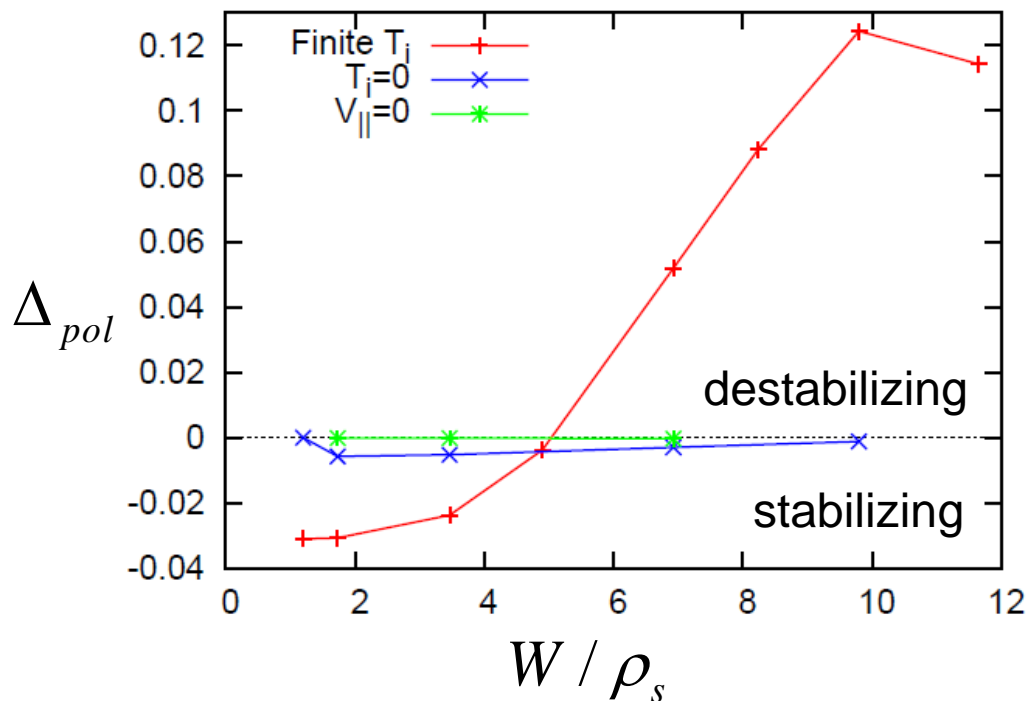
The sound wave propagation flattens the density within the island, so that the diamagnetic velocity is locally reduced (Scott, PoF 1985).



- Incompressible (Green line:  $V_{||} = 0$ )
  - The island is co-propagating with electrons.
- Cold ion plasma (Blue line:  $T_i = 0$ )
  - For small island width the propagation velocity is close to  $V_{*e}$ .
  - The velocity decreases as the width increases because of the density flattening.
- Hot ion plasma (Red line: Finite  $T_i$ )
  - For small island width the propagation velocity is close to  $V_{*e}$
  - The velocity decreases as the width increases, however the island never propagates in the ion direction.

# Polarization term $\Delta'$ becomes destabilizing for large island in hot ion plasmas

- Polarization current  $\Delta'$  is positive, i.e. destabilizing, in hot ion plasmas. This is in contrast to cold ion plasmas, where polarization current is negative.



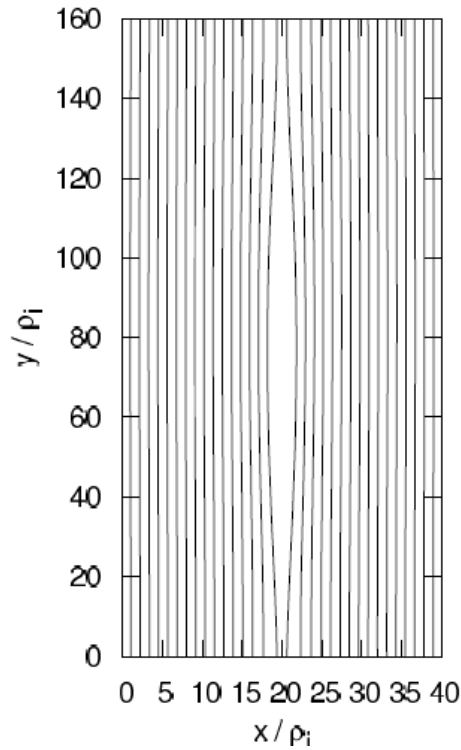
$$F_y(u = -V_{island}) = 0$$

$$\Delta_{pol} = - \int_0^{L_x} \frac{dx}{L_x} \int_0^{L_y} \frac{dy}{L_y} \frac{1}{\beta \Psi} J \cos k_{1y} y$$

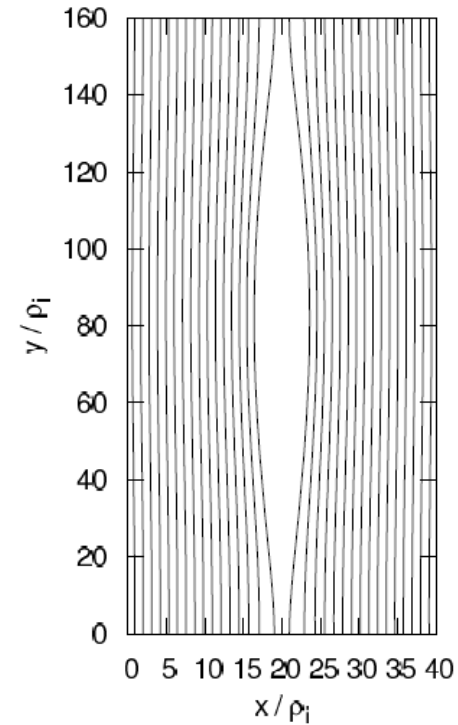
# Electron velocity (Finite Ti)

- Electrons are trapped within the island.

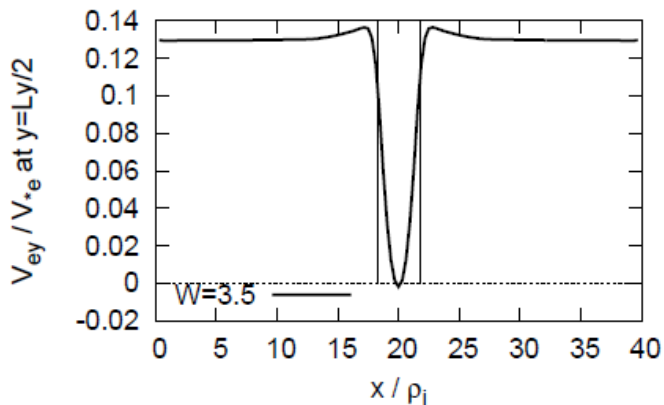
Finite  $T_i$ ,  $W=3.5$



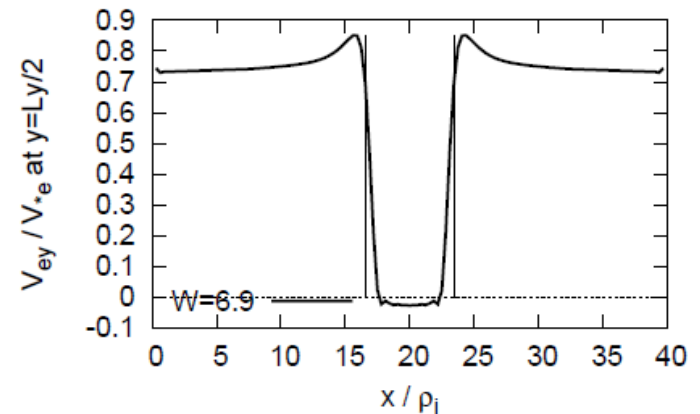
Finite  $T_i$ ,  $W=6.9$



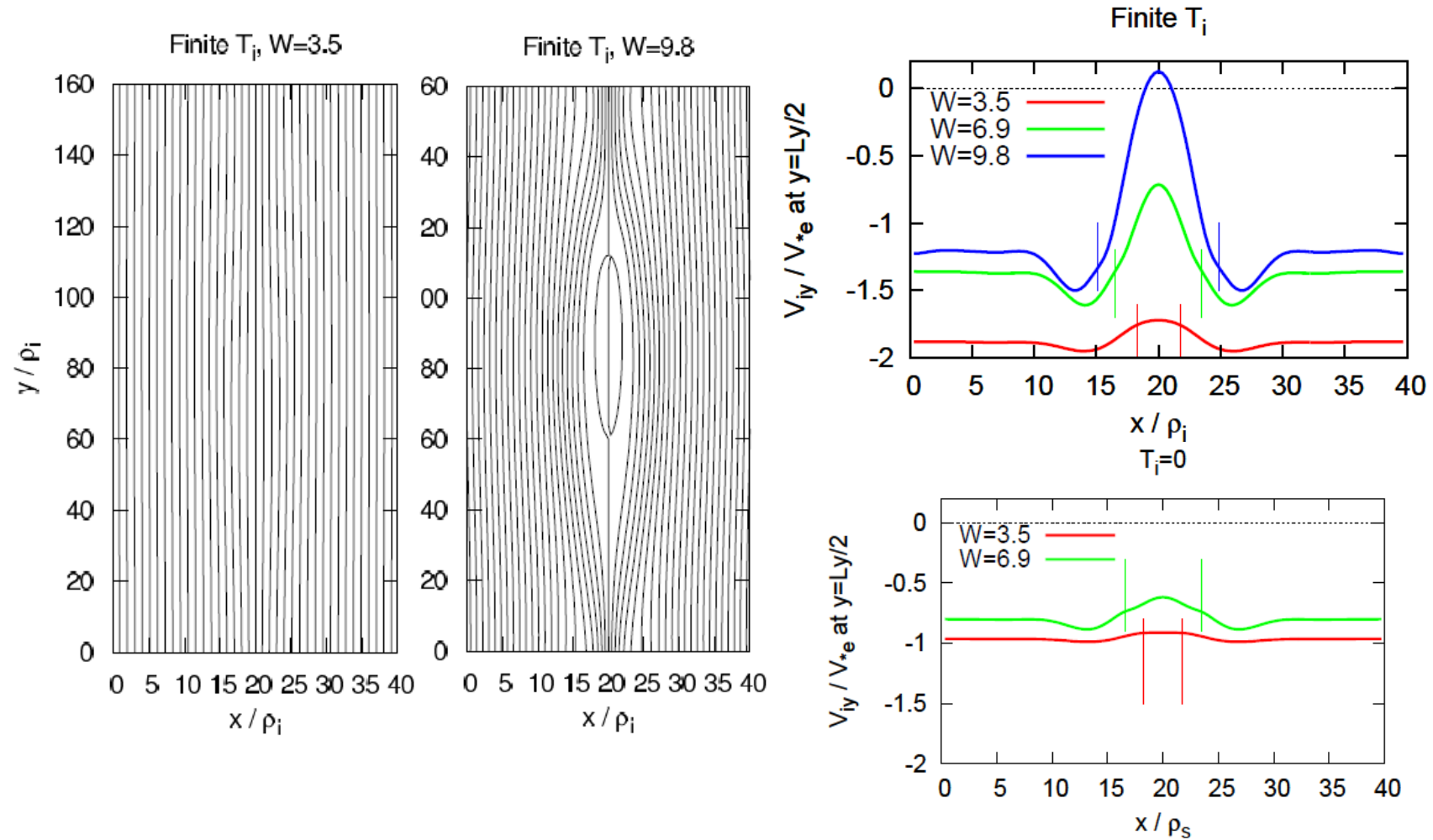
Finite  $T_{i0}$



Finite  $T_{i0}$



# Ion velocity (Finite $T_i$ )



- The large polarization current is due to the strong shear of the ion velocity around the separatrix of the islands.



# Summary

- The polarization current is found to be almost an order of magnitude larger in hot than in cold ion plasmas, due to the strong shear of ion velocity around the separatrix of the magnetic islands.
- Island propagation velocity
  - As a function of the island width, the propagation speed decreases from the electron drift velocity (for small islands) to the guiding-center velocity (for large islands).
- Polarization current
  - When the island width is larger than several times the ion Larmor radius, the polarization current is destabilizing (it drives island growth). This is in contrast to cold ion plasmas, where the polarization current is generally has a healing effect.