Extended MHD simulation of Rayleigh-Taylor/Kelvin-Helmholtz instability

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Background

- Numerical simulations using the single-fluid MHD model for high wave number ballooning instability[1]

- The single-fluid model: ignore Hall effect, Finite Larmor Radius (FLR) effect and so on.

- Since the single-fluid model can be insufficient for high wave number ballooning modes: use extended MHD model[2].

- Though extended MHD model is used as non-ideal MHD code such as M3D[3], NIMROD[4] and so on to explain experimental results, fundamental characteristics are not fully clarified especially in the nonlinear stage.

- Since the ballooning instability is Rayleigh-Taylor (RT) type instability, here we consider a simple RT instability.
Model and Method I

- 2D slab geometry
- Extended MHD equations
  (the Hall term and gyro-viscosity are added.)

\[
\frac{\partial \rho}{\partial t} = - \nabla \cdot (\rho \mathbf{v}) , \quad (1)
\]

\[
\frac{\partial}{\partial t} (\rho \mathbf{v}) = - \nabla \cdot \left[ \rho \mathbf{v} \mathbf{v} + \mathbf{I} \left( p + \frac{B^2}{2} \right) - \mathbf{B} \mathbf{B} + \delta \Pi_i \right] + \rho \mathbf{g} , \quad (2)
\]

\[
\frac{\partial e}{\partial t} = - \nabla \cdot \left[ \mathbf{v} (e + p) - \mathbf{v} \cdot \delta \Pi_i \right] , \quad (3)
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = - \nabla \cdot (\mathbf{v} \mathbf{B} - \mathbf{B} \mathbf{v}) - \nabla \times \left[ \frac{\epsilon}{\rho} (\mathbf{J} \times \mathbf{B} - \nabla p_e) \right] . \quad (4)
\]

\[
e = \frac{\rho \mathbf{v}^2}{2} + \frac{p}{\gamma - 1} \quad \epsilon : \text{Hall parameter} \quad \delta : \text{gyro-viscosity}
\]
model and method II

2D approximation:

\[ u_z = 0, \frac{\partial}{\partial z} \to 0 \quad \text{in eqs. (1)-(4)} \]

\[ J_x = \frac{\partial B_z}{\partial y}, \quad J_y = \frac{\partial B_z}{\partial x}, \quad J_z = \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \]

The gyro-viscosity term is given as

\[ (\Pi_i)_{xx} = - (\Pi_i)_{yy} = -p_i \left( \frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y} \right), \quad (5) \]

\[ (\Pi_i)_{xy} = (\Pi_i)_{yx} = p_i \left( \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} \right). \quad (6) \]

g : gravitational acceleration, \( \gamma \) : ratio of the specific heats

total pressure: \( p = p_i + p_e = (\alpha + 1)p_e \), \( \alpha = p_i/p_e \).

\( p_i, p_e \) : ion and electron pressure
Normalized Hall and gyro-viscosity parameter in Ref.[1]

Wave length: $\lambda=6\text{cm}$ when poloidal mode number $m=30$

Ion skin depth: $l_i = 5\text{cm}$  
($T=0.5\text{keV}, B=0.5\text{T}, n = 2 \times 10^{19}[\text{m}^{-3}]$)

Ion Larmor radius: $\rho_i = 6.5\text{mm}$

If $L = \text{minor radius} = 1.0[\text{m}]$

Gyro-viscosity

$$\delta = \frac{\rho_i}{L/2} = \frac{0.65[\text{cm}]}{50[\text{cm}]} \approx O(10^{-2})$$

Hall parameter

$$\epsilon = \frac{l_i}{L/2} = \frac{5[\text{cm}]}{50[\text{cm}]} \approx O(10^{-1})$$
Initial equilibrium in RT instability

- Equilibrium
  \[ \nabla \left( p_0 + \frac{B_0^2}{2} \right) = -\rho g \]
  \[ B_z = b_z + B_0(y) \]

- Numerical method
  \[ \rightarrow \text{4th order central difference} \]
  \[ \text{Time evolution : 4th order Runge-Kutta-Gill (RKG)} \]
- System size: \(-\pi \leq x \leq \pi, -3\pi \leq y \leq 3\pi\)
- Boundary condition: periodic \((x = \pm \pi), \partial / \partial y \rightarrow 0 (y = \pm 3\pi)\)
- Resolution: \((N_x, N_y) = (1024, 4086)\)
- Density ratio: \(\rho_2 / \rho_1 = 2.0\)
- \(\beta = 10\%\), density jump width: 0.2, \(p_i / p_e = 1.0\)
Initial perturbation: random perturbations are added

- Giving single-mode perturbation is time consuming

→ Add all perturbations in the initial condition.
- Random perturbed wave number: $k = 0 \sim N_x/4$ are added simultaneously.
• RT instability grows in time and forms turbulence-like structure.
• Since each modes are superimposed, mushroom-like structure doesn’t appear even in the linear stage.
Growth rate: suppression of the higher modes

- Linear growth rate is evaluated by the gradient of the integrated kinetic energy.
- In this parameter, combination of the Hall term and the gyro-viscosity term stabilize the growth rate of the higher wave number modes.
High wave number modes are suppressed and only low wave number modes grow.

Density plot: low wave number modes grow

MHD

Hall=0.30, Gyro=0.03

- t=15.0
- t=20.0
- t=25.0

- High wave number modes are suppressed and only low wave number modes grow.
Initial Condition in KH instability

\[ U_0 = 0.1 - 0.1 \, \rho \]

\[ \rho_2 = 4.0 \]

\[ \rho_1 = 1.0 \]

\[ \nu = 0.02\pi \]

\[ \delta = 0.01 \pi \]

\[ U_0 = \begin{cases} 0.1 & \text{for } \rho_1 \\ -0.1 & \text{for } \rho_2 \end{cases} \]

\[ (N_x, N_y) = (256, 1024) \]
Result

\[ U_0 = -0.1 \quad \leftrightarrow \quad U_0 = 0.1 \]

density contour of (1-fluid) MHD

![Graph showing growth rate vs. wave number for different models.](image)

Legend:
- Linear MHD
- Hall 0.5
- Gyro 0.05
- Hall&Gyro
Result

\[ U_0 = -0.1 \quad \leftrightarrow \quad U_0 = 0.1 \]

The Hall and FLR effects differ depending on the flow direction.

The density contour of (1-fluid) MHD shows variations depending on the flow direction and the value of \( U_0 \).
On the Hall effect, the relation is qualitatively consistent with the result of linear analysis.

\[ U_0 = -0.1 \]

\[ U_0 = 0.1 \]
Summary I (RT instability)

- The effects of the Hall term and the gyro-viscosity to the RT and KH instabilities are studied by the nonlinear extended MHD simulations.
- Linear region has the same time scale when random perturbed modes are added simultaneously.
- In RT simulation, the Hall term slightly increase and the gyro-viscosity decrease the growth rate in our parameter. This result is consistent with the result by Winske[5].
- Combination of the Hall term and the gyro-viscosity highly stabilize the linear growth rate of the high wave number modes and only low wave number modes grow.
Summary II (KH instability)

• The Hall term can stabilize KH modes, especially in high wave number.
• The dispersion relation obtained from this simulation is qualitatively consistent with the result of linear analysis on the Hall effect.
• The gyro-viscosity can destabilize KH modes.
Future plan

- Simulate both the RT instability and the KH instability simultaneously.
- Extend the geometry to a 3D torus system to analyze the evolution of the ELMs.

2D hollow cylinder $\rightarrow$ 3D hollow cylinder $\rightarrow$ 3D torus

Reference