Turbulence near the reverse surface measured by microwave imaging reflectometry on TPE-RX


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Microwave Imaging Reflectometry (MIR)

- Motivation: 2D imaging of the fluctuation, just like a photograph.
- MIR is based on the radar technique
  - Microwave is reflected at the cutoff surface, we can get the phase difference
  - The phase fluctuation is dominated by the density fluctuation close to the cutoff surface
    \[ \delta \phi \propto \delta n_e (r_{cut}) \]
  - Local measurement
- However, the fluctuation is 2D or 3D which lead to interference at the detector surface
- The optical lens can restore phase front
- MIR uses large-lens optical imaging.
  - Experimental verification, TEXROR, LHD,...
  - Simulations (Princeton PPL and KASTEC)
TPE-RX

- TPE-RX: $R / a = 1.72m / 0.45m$.
- TPE-RX: Reverse field pinch
- RFP device is characteristic of a reverse toroidal field in the outer region.
- $B_\phi \sim B_\theta$, $q<<1$.
- RFP configuration is obtained as a result of MHD relaxation and sustained by dynamo-effect.

- Plasma current: $I_p \sim 300$ kA
- Plasma density: $n_e \sim (0.6-1.0) \times 10^{19}$ m$^{-3}$
- Pinch parameter: $Q = B_{pw}/<B_t> \sim 1.5-2$
- Reversed parameter: $F = B_{tw}/<B_t> \sim -0.16-1$
MIR system on TPE-RX

- **O-mode** is used ($B_0 >> B_\phi$, $\rho > 0.5$)
- The cutoff density: $n_c = 0.5 \times 10^{19} \text{m}^{-3}$
- Spatial resolution: **3.7cm**, Time resolution: **1\(\mu\)s / 0.5\(\mu\)s.
The electron density profile is measured using a dual-chord interferometer and the normalized radius are 0.0 and 0.69, respectively.

Electron density profile is obtained by

\[ n_e(r) = n_e(0)(1-r^4)(1+Ar^4) \]

\( A \) is the profile factor

The typical plasma density profile is flat or hollow. If \( n_e(r) > n_c \), the cutoff surface is at the plasma edge.

- The cutoff surface is close to the reverse field surface during PPCD.
- The large mode numbers near by the reverse surface.
**Pulsed Poloidal Current Drive (PPCD) plasma**

- The soft-X-ray intensity increases rapidly during PPCD.
- The cutoff radius is about 0.9.
- The amplitude of the density fluctuation increases during PPCD.

**Lissajous’ curve of I-Q signals**

- IQ: sine and cosine components of density fluctuations.
- IQ circle: fluctuation of the cutoff.
- One circle: 1.5cm.
The decay index \( (f^{-a}) \) of the power spectrum represents a qualitative indication of an energy exchange process between fluctuations at different scales.

The spectrum shows three distinct frequency ranges, each with characteristic power dependence.

- **Power spectrum**
  - The power spectrum shows the power law decay like \( f^{-2} \) in the frequency range 70-200 kHz and \( f^{3-4} \) in the frequency of 300-700 kHz.
The phase shift has linear tread at $f<200\text{kHz}$, which means a constant phase velocity.

But at $f>200\text{kHz}$, the phase almost doesn't change which means the fluctuation has a constant wavenumber.

At low $k$ ($k<16\text{m}^{-1}$), phase velocity keeps constant (about 50km/s).

The velocity is about 3 times lower than the ion sound speed.

The phase velocity increase rapidly at high $k$. 
Radial scanning by changing plasma density. The cutoff moves to outside with the increase of density, close to the reverse surface. The phase velocity sharply decreases at \( \rho \sim 0.9 \) which suggests a velocity shear around the reverse region.
The fluctuation is dominated by $f < 200\text{kHz}$ and $k < 20/m$.

The spectrum shows linearly trend which means a constant phase velocity at low $k$ fluctuation.

It becomes saturated at high $k$ fluctuation.

The fluctuations propagate in the electron drift direction.
The correlation length is obtained by averaging cross-correlation over different pairs of channels.

- The toroidal correlation length is defined as the separation for which the coherence has decreased to 1/e.
- The toroidal correlation length has a negative correlation with the wavenumber (k).
  - The correlation length is about 10-30cm for low k fluctuation.
  - The correlation length is about 3cm for high k fluctuation.
- The fluctuations at different scales have different coherence length, which suggest multi-wave in PPCD plasma.
The definition of entropy is given as:

$$H = \int \ln \sum_n R_x[n] e^{-j\omega n} d\omega$$

$R_x$ is the Fourier transform of the cross-correlation function.

Generally, the detector size is smaller than the correlation length.

It is the similar as a filter used in the cross-correlation matrix.

$$\sum_{(i,j) \in B} a_{ij} R_x(r-i,s-j) = R_x(r,s) \quad \text{for } (r,s) \in B$$

where, $a_{ij}$ is the autoregressive filter coefficients to be estimated.

Iteratively solve for $a_{ij}$:

$$\hat{p}_x(\omega_1, \omega_2) = \frac{1}{\sum_{(k,l) \in B} e^{-j\omega_1 k} e^{-j\omega_2 l}}$$

The correlation matrix is obtained by averaging cross-correlation over different pairs of channels.

Difference

Max Entropy method

- Image is extrapolated using high resolution 2D power spectrum estimation techniques
- Max Entropy extends correlation measurements outside measured region

FFT method

- Size of image in plasma is limited by narrow port.
- Coherence does not go to zero
- Broadening is due to finite beam width

\[ \gamma \propto \frac{1}{w} \]

Image is extrapolated using high resolution 2D power spectrum estimation techniques.

Max Entropy extends correlation measurements outside measured region.

\[ \gamma \propto \frac{1}{w} \]
power spectrum

Real part of cross-correlation

Imag. part of cross-correlation

Traditional FFT method

Lim & Malik MEM
Power spectrum estimated by Lim & Malik MEM

Shot, 53330, PPCD, Rcut=0.75-0.9, f=290±20kHz

- Measurement ranges:
  \(|k| < 85\text{m}^{-1},\)
  \(|m|/|n| < 34/146.\)

- The \(k\) is normalized by 0.037/\(\pi\).

- Multi- modes are obtained by MEM. Note that some of them might be caused by the aliasing or the power is too low.

- The high toroidal \(k\) is poloidally elongated due to too less channels in poloidal direction.
Reflective signal can be significantly influenced by the waves away from the cutoff layer, whose amplitude is sufficiently large. → the spatial resolution becomes poor.
Fluctuation structure

Skewness (S) and Kurtosis (K):

\[ S = \frac{\langle \tilde{x}^3 \rangle}{\langle \tilde{x}^2 \rangle^{3/2}} \]
\[ K = \frac{\langle \tilde{x}^4 \rangle}{\langle \tilde{x}^2 \rangle^2} - 3 \]

- The skewness and kurtosis are used to describe the asymmetry and peak degrees of a distribution with respect to its mean value, respectively.
- For a Gaussian random distribution, \( S = K = 0 \).
- There is no interaction of ambient turbulence if \( S = K = 0 \) for all frequency components.
- A non-Gaussian tail means the existence of coherent structures or intermittency of ambient turbulence.

- The fluctuations at different frequencies are identified by the band pass filter (BPF)
- The fluctuation shows positive bursts at low frequency (\( f < 150 \text{kHz} \)) and negative bursts at high frequency (\( f > 150 \text{kHz} \)).
- The dominant burst frequency is <600kHz
Three wave coupling

Similar trends appear in the bicoherence and kurtosis.

The kurtosis and skewness have reverse trends.

The bicoherence has a broad profile between 200-500kHz, which represents the existence of nonlinear coupling ($k \sim 20 \text{m}^{-1}$).

The bicoherence has stronger dependence on the skewness and the kurtosis.

The local peaks of bicoherence are about 300kHz, 400kHz and 800kHz.

The local peak of skewness is about 300kHz while the local peak of kurtosis is about 180kHz.

The different local peaks suggest existence of nonlinear multi-wave interaction.
Two-dimensional electron density fluctuation have been measured by microwave imaging reflectometry (MIR) on TPE-RX.

The measurements confirm several important properties of plasma edge turbulence, such as velocity shear, k and intermittency.

The phase velocity is about 50km/s at low k fluctuations, which propagate in the electron drift direction.

The fluctuation structures are quantified by skewness and kurtosis.

The power spectrum is estimated by the Lim & Malik maximum entropy method (MEM), which shows the existence of multi-modes close to reverse region.

The m=0 and m=1 modes has a reverse toroidal propagation direction

The different local peaks of bicoherence, skewness and kurtosis suggest nonlinear multi-wave interactions.
Thank you for your attention !
Malik MEM has sidelobe problem and convergence problem. It gives spurious peaks and distorts the spectrum sometimes.

Small difference in the toroidal direction, but large difference in the poloidal direction.

Skilling MEM shows a continuous and a more reliable spectrum. But it fails to solve the low k modes.
Quasi-single helicity (QSH) plasma

The quality of the QSH state can be estimated by

\[ N_s = \left[ \sum_n \left( \frac{W_n}{\sum_{n'} W_{n'}} \right)^2 \right]^{-1} \]

where, \( W_n \) is the energy of the \((m=0 \text{ or } 1)\) mode.

- \( N_s = 1 \) SH state
- \( N_s < 3 \) QSH state
- \( N_s > 3 \) MH state

- \( N_s = 2 \) at 35-47ms.
- The dominate mode is \( m=1, n=6 \) during QSH state
- The cutoff radius is about 0.75 during QSH
The fluctuations versus frequency are identified by the band pass filter.
QSH shows a more intermittent behavior (around 200-400 kHz).