Relation between poloidal ion flow and magnetism of toroidal field in multi-pulsing CHI driven ST

T. Kanki\textsuperscript{1} and M. Nagata\textsuperscript{2}

\textsuperscript{1}Japan Coast Guard Academy, \textsuperscript{2}University of Hyogo

NEXT Workshop
10 - 11 March, 2016; Kyoto Terrsa, Japan

Outline:
1) Introduction: Background and purpose
2) Double-pulsing CHI experiment on HIST:
   Radial profile (flow velocity, number density, and radial electric field)
3) Axisymmetric two-fluid equilibrium equations
4) Numerical results: Relation between poloidal ion flow velocity and toroidal field
5) Summary
Introduction

- **Coaxial helicity injection (CHI):** Non-inductive CHI and inductive CHI
  - Non-inductive CHI has been used as plasma start-up (Transient CHI) and quasi-steady state current drive (Driven CHI) in a spherical torus (ST)
  - In the driven phase, the fluctuations deteriorate the confinement.
  - In the decay phase, the closed flux surfaces are formed, resulting in the good confinement.

- **Multi-pulsing CHI (M-CHI):** after the plasma current partially decays, a new CHI pulse is applied and the cycle process is repeated.
  - To achieve a quasi-steady sustainment and good confinement

- **Double-pulsing CHI experiment in the HIST device**
  - Observation around the Central open flux column (OFC) region
    - Steep density gradient (width of OFC $w_{OFC} \sim 7\text{cm}$, ion skin depth $\ell_i \sim 3\text{ cm}$)
    - Poloidal shear flow and the radial electric field shear
    - *Diamagnetic toroidal field*
    - *Two-fluid effect* is important.

- Two-fluid flowing equilibrium
  - *Relation between poloidal ion flow and toroidal field caused by applying the CHI pulse again*

Radial profiles in double-pulsing CHI experiment on HIST

The poloidal flow velocity is increased around the separatrix, and its shear is enhanced there.

The toroidal flow velocity is increased in the closed flux region.

The density gradient and the radial electric field shear is enhanced around the separatrix.
Normalized non-dissipative two-fluid equations in a steady state

Equation of ion motion
\[ u \cdot \nabla u = -\nabla p_i / n + E + (1/\varepsilon)u \times B \]

Equation of electron motion
\[ 0 = -\nabla p_e / n - E - (1/\varepsilon)u_e \times B \]

Equations of continuity
\[ \nabla \cdot (nu) = 0 \quad \nabla \cdot (nu_e) = 0 \]

Entropy conservation
\[ u \cdot \nabla s_i = 0 \quad u_e \cdot \nabla s_e = 0 \]

Equations of state
\[ p_i = n^\gamma \exp[(\gamma - 1)s_i] \quad p_e = n^\gamma \exp[(\gamma - 1)s_e] \]

Gauss’ law for magnetic field
\[ \nabla \cdot B = 0 \]

Ampere’s law
\[ n(u - u_e) = \varepsilon \nabla \times B \]

Faraday’s law
\[ \nabla \times E = 0 \quad \Rightarrow \quad E = -\nabla \phi_E \]

Two-fluid parameter: \( \varepsilon \equiv \ell_i / L \)

ion skin depth: \( \ell_i \equiv c / \omega_{pi} \), \( L \): system scale length
\[ \ell_i \propto m_i^{1/2} \quad \Rightarrow \quad \varepsilon : \text{ion inertial effect} \]

HIST \( \varepsilon = 0.072 \)
NSTX \( \varepsilon = 0.034 \)
TS-3(FRC) \( \varepsilon = 0.20 \)
### Axisymmetric two-fluid equilibrium equations

**Generalized Grad-Shafranov equations for ion and electron surface variables**

**Ion:**

\[
\overline{v}_i' r^2 \nabla \cdot \left( \frac{\overline{v}_i' \nabla Y}{n \ r^2} \right) = \frac{r}{\varepsilon} \left( B_\theta \overline{v}_i' - n u_\theta \right) + n r^2 \left( H_i' - T_i S'_i \right) \rightarrow u_\theta
\]

- Poloidal flow inertia
- \(u \times B\) force
- Gradient of pressure, flow energy, and electrostatic potential

**Electron:**

\[
r^2 \nabla \cdot \left( \frac{\nabla \psi}{r^2} \right) = \frac{r}{\varepsilon} \left( B_\theta \overline{v}_e' - n u_\theta \right) - n r^2 \left( H_e' - T_e S'_e \right) \rightarrow \psi
\]

**Generalized Bernoulli equations for density**

**Ion:**

\[
\frac{\gamma}{\gamma - 1} n^{\gamma - 1} \exp\left[ (\gamma - 1) S_i \right] + \frac{u^2}{2} + \phi_E = H_i
\]

- Enthalpy

**Electron:**

\[
\frac{\gamma}{\gamma - 1} n^{\gamma - 1} \exp\left[ (\gamma - 1) S_e \right] - \phi_E = H_e
\]

**Arbitrary surface functions**

- \(\overline{v}_e(\psi), \overline{v}_i(Y)\)
- \(H_e(\psi), H_i(Y)\)
- \(S_e(\psi), S_i(Y)\)

**Auxiliary equations**

\[
Y(r,z) = \psi + \varepsilon r u_\theta
\]

\[
B_\theta = \frac{1}{\varepsilon r} \left( \overline{v}_i - \overline{v}_e \right)
\]

\[
u_p = \frac{\nabla \overline{v}_i}{nr}
\]

\[
T_\alpha = n^{\gamma - 1} \exp\left[ (\gamma - 1) S_\alpha \right]
\]
**Boundary condition and assumption**

Free slip boundary condition is imposed for toroidal ion flow velocity $u_\theta$

Function form of bias flux

$$\psi_{\text{bias}}(r) = \frac{4\psi_s}{(R_1 - R_2)^2} (r - R_2)(R_1 - r)$$

- **Assumption**
  - The poloidal electron flow along the open field lines is mainly driven by applying the CHI pulse.

  I change to decrease the poloidal flow velocity in the OFC region.
The toroidal ion flow velocity is increased in the closed flux region due to the increase of the electron drift velocity and the Hall effect.

Generalized Grad-Shafranov equation for electron

\[ u_\theta = \frac{\overline{\psi_e}'}{n} B_\theta - \epsilon r \left( H_e' - T_e S_e' \right) - \epsilon \frac{r}{n} \nabla \cdot \left( \frac{\nabla \psi}{r^2} \right) \]

- MHD term
- Electron drift velocity
- Hall effect

- The MHD term is almost cancelled by the Hall term.
- The electron drift velocity term survives, causing the toroidal ion flow velocity in the closed flux region.
The ion flow energy increases with the toroidal ion flow velocity in the closed flux region.

The increase in the ion flow energy causes the decrease in the enthalpy in accordance with the generalized Bernoulli law.

The decrease in the enthalpy causes the drop in the density.

The density is decreased in the closed flux region, and it gradient steepens around the separatrix in the high field side.

\[
\begin{align*}
\underbrace{h_i + h_e + \frac{u^2}{2}}_{\text{Enthalpy}} &= H_i + H_e, \\
\underbrace{h_{\alpha}}_{\text{Ion flow energy}} &= \frac{\gamma}{\gamma - 1} n^{\gamma-1} \exp[(\gamma - 1)S_{\alpha}] \\
\underbrace{n}_{\text{Density}}
\end{align*}
\]
The density is decreased in the closed flux region and its negative gradient around the separatrix steepens.

The ion temperature is slightly increased in the closed flux region, and becomes to the broad profile.

In the electron temperature, the hollow profile is enhanced.

The gradient of the ion pressure becomes negative around the separatrix.

In the electron pressure, the hollow profile is enhanced.
Radial electric field and two-fluid effect

The radial electric field is enhanced around the separatrix.

The two-fluid effect is due to the ion diamagnetic effect.

The radial electric field strongly depends on the magnetic force $1/\varepsilon u_t B_z$.

Ohm's law:

$$E + \frac{1}{\varepsilon} u \times B + F_{2F} = 0,$$

Two-fluid effect:

$$E_r + \frac{1}{\varepsilon} (u_t B_z - u_z B_t) + F_{2Fr} = 0$$

Ion diamagnetic effect

Ion inertial effect
Poloidal component of drift velocity

**Case I**
- The ion diamagnetic drift velocity is comparable to the $ExB$ and electron diamagnetic ones, but the ion inertial drift velocity is small.

**Case II**
- The ion diamagnetic drift velocity is increased around the separatrix, changing the same direction as the the $ExB$ one there.
  - ➡️ increase in the poloidal flow velocity

**Case III**
- The $ExB$ drift velocity is decreased in the closed flux region, changing the opposite direction to the ion diamagnetic drift velocity there.
  - ➡️ decrease in the poloidal flow velocity
  - ➡️ increase in the flow shear

**Graphs**
- $E \times B$ drift velocity: $u_E = \frac{eE}{B^2}$
- Diamagnetic drift velocity: $u_D = \frac{eB \times \nabla p_i}{B^2 n}$
- Inertial drift velocity: $u_i = \frac{eB \times (u_i \cdot \nabla u_i)}{B^2}$
Flow velocity profiles at the midplane

**Toroidal flow velocity**

*✓* The ion flow velocity is increased from negative to positive values in the closed flux region, enhancing the flow shear and contributing to the increase in the current.

*✓* The electron flow velocity is slightly decreased in the OFC region.

**Poloidal flow velocity**

*✓* The ion flow velocity is increased in the OFC region, enhancing the flow shear around the separatrix, and contributing to the increase in the current.
The strength of diamagnetic ion current density is increased in the closed flux region, approaching that of diamagnetic electron current density.

The total diamagnetic current is carried by both ion and electron fluids.

The total current profile becomes steep in the closed flux.

The paramagnetic toroidal field is generated in the closed flux region.
The toroidal magnetic field becomes from a **diamagnetic** to a **paramagnetic** profile in the closed flux region, while the diamagnetic profile is kept in the OFC region.

The toroidal current density and $\lambda$ have the **hollow profiles**, and are increased in the closed flux region.
Summary

We have investigated the relation between poloidal ion flow velocity and toroidal field due to applying the CHI pulse again.

- The CHI pulse causes the steep density gradient around the separatrix in the high field side.

- The ion diamagnetic drift velocity is changed to the same direction as the the ExB one around the separatrix as the ion pressure gradient steepens there. The ExB drift velocity is decreased in the closed flux region, changing the opposite direction to the ion diamagnetic drift velocity there.

- The poloidal ion flow velocity is increased around the separatrix but decreased in the closed flux region, enhancing the flow shear.

- The toroidal magnetic field becomes from a diamagnetic to a paramagnetic profile in the closed flux region, because the poloidal diamagnetic current profile becomes steep there.