Recent progress of toroidal full-$f$ gyrokinetic simulation based on GKNET

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Local/Global Gyrokinetics

**Local approach**

\[ \partial_t f_{eq} - [H, f_{eq}] = C(f_{eq}) + S \]

\[ \partial_t \delta f - [H, \delta f] - [\delta H, f_{eq}] - [\delta H, \delta f] = C(\delta f) \]

- Linear
- Driving
- Nonlinear

**Global approach**

\[ \partial_t f_{eq} - [H, f_{eq}] = C(f_{eq}) + S \]

- Self-consistently determined Mean $E_r$

\[ \partial_t \delta f - [H, \delta f] - [\delta H, f_{eq}] - [\delta H, \delta f] = C(\delta f) \]

- Linear
- Driving
- Nonlinear

**Fixed Gradient**

\[ \frac{R}{L_T} \neq 0 \]

\[ T = \text{const} \]

- Heat/particle sink
- Heat source

**Global profile shear effect** can be taken into account (e.g. $\omega_r$ shear)

- Very powerful tool to estimate turbulent transport process

- Computationally efficient
  - -> multi-species, EM turbulence

**Fixed Flux**

\[ \frac{R}{L_T} \neq 0 \]

\[ T \neq \text{const} \]

- Mean $E_r$ is self-consistently determined
  - -> ITBs, L-H transition...

**GKV(JPN), GS2(US), GENE(GER), ...**

**GT5D, GKNET(JPN), XGC(US), GYSELA(FRA), ...**
GK Vlasov equation for ion

\[ \frac{\partial f}{\partial t} + \frac{dR}{dt} \cdot \frac{\partial f}{\partial R} + \frac{dv_\parallel}{dt} \frac{\partial f}{\partial v_\parallel} = C_{coll} \]

\[ \frac{dR}{dt} \equiv \{R, H\} = v_\parallel b + \frac{c}{eB^*_\parallel} b \times (e\nabla\langle\phi\rangle_\alpha + m_i v^*_\parallel b \cdot \nabla b + \mu \nabla B) \]

\[ \frac{dv_\parallel}{dt} \equiv \{v_\parallel, H\} = -\frac{B^*_\parallel}{m_i B^*_\parallel} \cdot (e\nabla\langle\phi\rangle_\alpha + \mu \nabla B) \]

Vlasov solver
✓ 4th-order Morinishi scheme + 4th-order RK-Gill scheme

GK quasi-neutrality condition

\[ \phi - \langle\phi\rangle_\alpha + \frac{1}{T_{e0}(r)} (\phi - \langle\phi\rangle_\alpha) = \frac{1}{n_{i0}(r)} \int \int \langle\delta f\rangle_\alpha B^*_\parallel dv_\parallel d\mu \]

Real space field solver
✓ Full-order FLR effect (without Tayler/Pade approximation)
✓ Field equation is solved in real space (not k-space)

Recent Progress Based on GKNET

**Study of flux-driven ITG turbulence**

(A-1) Flux-driven turbulent transport couple with mean flow
   [Y. Kishimoto, et al., submitted to IAEA-2016]
   [W. Wang, et al., this workshop]
   - Global profile shear effect of $\omega_r$ and $\omega_f$ on ballooning structure
   - Intermittent turbulent transport coupled with radially extended ballooning structure

(A-2) ITB formation in flux-driven turbulence
   [K. Imadera, et al., submitted to ICPP-2016 & IAEA-2016]
   [S. Maeda, et al., this workshop]
   - ITB formation by toroidal momentum injection
   - Momentum pinch originated from global profile shear effect of $\omega_r$ and $\omega_f$

**Development of GKNET**

(B-1) Development of real space field solver
   [K. Obrejan, et al., this workshop]
   - Elongation $\uparrow$ or triangularity $\uparrow$
     - Residual zonal flow level $\uparrow$

   ![Graph showing the relationship between elongation and triangularity and residual zonal flow level](image)

   - Kinetic electron slab ITG instability $\searrow$
   - Phase shift between $\phi$ and $\delta n_e$

(B-2) Introduction of kinetic electron
   [R. Yoshida, et al., this workshop]
   - Kinetic electron $\rightarrow$ slab ITG instability $\searrow$
   - Phase shift between $\phi$ and $\delta n_e$
   - Momentum pinch originated from global profile shear effect of $\omega_r$ and $\omega_f$ $\rightarrow$ residual ZF level $\uparrow$
Profile stiffness is a long standing problem, which may limit the overall performance of H-mode plasmas.

In the JET experiment, while strong temperature profile stiffness is observed, it can be greatly reduced by co-current toroidal rotation in weak magnetic shear plasma.

In our flux-driven ITG simulation, we also observe a stiff temperature profile in the absence of momentum source, where not only heat avalanches but also the explosive global transport coupled with the instantaneous formation of radially extended ballooning structure become dominant.

A) Why radially extended structure is formed even in the presence of MF and ZF?
B) What is the stabilization mechanism by co-current toroidal rotation?
Purpose of This Work

**Purpose of this work**

A) Understand the origin of radially extended ballooning structure in flux-driven ITG turbulence with MF and ZF -> **profile stiffness**

B) Control such structures by momentum injection -> **barrier formation**

**Approaches**

1. Non-local first-order ballooning theory
   - Notation of $\theta_b$, $\Delta r$ and $\gamma$
   - Impact of MF and toroidal rotation on toroidal ITG mode

2. Global GK ITG simulation w/o mom. source
   - Impact of MF on profile stiffness

3. Global GK ITG simulation with mom. source
   - Impact of momentum injection on profile stiffness
Non-Local Ballooning Theory

$\theta_b = \pm \left| \frac{\partial_r (\omega_r + \omega_f)}{2k_\theta \gamma_0 \hat{s}} \right|^{1/3}$

$\Delta r = \left| \frac{\sin \theta_b}{k_\theta \hat{s}^2 \theta_b^3} \right|^{1/2}$

$\gamma = \gamma_0 \cos \theta_b$

Radial force balance

$E_r - v_\theta B_\phi + v_\phi B_\theta - \frac{1}{n_i e} \frac{\partial p_i}{\partial r} = 0$

$E_r = \frac{r B}{q R} U_\parallel - \frac{T_i}{e} \left( \frac{1}{L_n} + \frac{1 - k}{L_{Ti}} \right)$

$n_i = n_{i0} \exp \left( - \frac{r}{L_n} \right), T_i = T_{i0} \exp \left( - \frac{r}{L_{Ti}} \right)$

Eigenfrequency + Doppler shift frequency

$\omega_r + \omega_f \sim \frac{k_\theta}{eB} \left[ \left( \frac{2}{R_0} - \frac{1}{L_n} - \frac{1 - k}{L_{Ti}} \right) T_i - \frac{e r B}{q R} U_\parallel \right]$

✓ Cancellation by mean flow
✓ Impact of toroidal rotation

Diamagnetic drift  Mean flow  Toroidal rotation

**Simulation condition**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>$a_0/\rho_i$</td>
<td>300</td>
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<tr>
<td>$a_0/R_0$</td>
<td>0.36</td>
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<td>$(R_0/L_n)_{r=a_0/2}$</td>
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<tr>
<td>$(R_0/L_{T_i})_{r=a_0/2}$</td>
<td>6.92</td>
</tr>
</tbody>
</table>

**Numerical results**

- **Without $E_r$**
  - $\gamma$: $0.07 \sim 0.12$
  - $\theta_b$: $0.5 \sim 0.6$
  - $\Delta r$: $28 \sim 42$

- **With $E_r$**
  - $\gamma$: $0.15$
  - $\theta_b$: $0$
  - $\Delta r$: $49$

\[ \theta_b = \mp \sqrt[3]{\frac{\partial r(\omega_r + \omega_f)}{2k_\theta \gamma_0 \hat{s}}} \]
Nonlinear Flux-Driven GK ITG Simulation

**Simulation condition**

- Safety factor $q$, $\hat{s}$
- Ion temperature $T_i$
- Electron temperature $T_e$
- Ion density $n_i$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>$a_0/R_0$</td>
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<td>$(R_0/L_{T_e})_{r=a_0/2}$</td>
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<tr>
<td>$\nu_*$</td>
<td>0.28</td>
</tr>
<tr>
<td>$P_{in}$</td>
<td>4, 8, 16, 24 [MW]</td>
</tr>
</tbody>
</table>

**Source operator**

$$S_{src} = A_{src}(r) \tau_{src}^{-1} [f_M(2\bar{T}) - f_M(\bar{T})]$$

- Constant power input near magnetic axis

**Sink operator**

$$S_{snk} = A_{snk}(r) \tau_{snk}^{-1} [f(t) - f(t = 0)]$$

- Krook-type operator to $f$ in boundary region

[Y. Idomura, et. al., Nucl. Fusion, 49, 065029 (2009).]
Poloidal Symmetry and Profile Stiffness

2D spatial correlation analysis for potential structure (16MW)

Ballooning angle is smaller than that estimated from linear analysis without $E_r$.

Gradient-Flux relation in power scan test

Nonlinear critical threshold
Discussion - How we can break profile stiffness?

✓ Mean flow shear recovers the symmetry or weakly reverses the ballooning angle so that its stabilization effect is small.

✓ Toroidal rotation can change the mean flow shear through radial force balance, by which we may enhance its stabilization effect.

✓ Especially, toroidal rotation in outer region with small safety factor (weak/reversed magnetic shear) can be effective.
Flux-Driven ITG Simulation with Momentum Source

Simulation condition

- Magnetic shear $\hat{s}$
- Electron temperature $T_e$
- Ion density $n_i$
- Ion temperature $T_i$

Parameter Value

- $a_0/\rho_i$: 150
- $a_0/R_0$: 0.36
- $(R_0/L_n)_{r=a_0/2}$: 2.22
- $(R_0/L_{T_i})_{r=a_0/2}$: 10.0
- $(R_0/L_{T_e})_{r=a_0/2}$: 6.92
- $v_*$: 0.28
- $P_{in}$: 4 [MW]
- $T_{in}$: 11.2 [N·m]

Momentum source operator

$$S_M = \tau_M^{-1} A(r)[f_{LM}(n_0, 0.5v_{ti}, T_0) - f_{LM}(n_0, 0, T_0)]$$

$$f_{LM}(n, U||, T) = \frac{n}{\sqrt{2\pi T^3/m_i^3}} \exp\left[ -\frac{0.5(v|| - U||)^2 + \mu B}{T/m_i} \right]$$

We compare two cases;

(A) without momentum source

(B) with momentum source at $r = 90\rho_i$
Strong impact of momentum source at outer region on temperature build-up.

(A) No momentum source

(B) Source at $r = 90 \rho_i$

$\frac{T_i}{T_0}$ vs. $\frac{r}{\rho_i}$
Impact of Momentum Source - 2

Radial force balance: \[ E_r + \frac{k}{e} \frac{\partial T_i}{\partial r} - \frac{rB}{qR} U_\parallel - \frac{1}{n_i e} \frac{\partial p_i}{\partial r} = 0 \]

1. Strong correlation
   - No momentum source
   - Source at \( r = 90 \rho_i \)

2. Impact of Momentum Source - 2
   - No momentum source
   - Source at \( r = 90 \rho_i \)

3. Impact of Momentum Source - 3
   - No momentum source
   - Source at \( r = 90 \rho_i \)

4. Impact of Momentum Source - 4
   - No momentum source
   - Source at \( r = 90 \rho_i \)
Impact of Momentum Source - 3

Strong $E_r$ shear triggered by toroidal rotation in outer region suppresses the turbulence, leading to a transport barrier formation.

[M. Kikuchi and M. Azumi, Rev. Mod. Phys. 84, 1807 (2012).]
Only co-current toroidal rotation can benefit the ITB formation in weak magnetic shear plasma.

→ qualitative agreement with the observations in the JET experiment
We have newly developed 5D toroidal full-f gyrokinetic code *GKNET*.

We found that a momentum source can change the mean $E_r$ through the radial force balance, leading to ITB formation.

The underlying mechanism is identified to originate from a positive feedback loop between the enhanced mean $E_r$ shear and resultant momentum pinch, which can be observed only in co-input case.
Future Plans

Flux-driven turbulent transport couple with mean flow

ITB formation in flux-driven ITG turbulence

Introduction of kinetic electron

Magnetic shaping effect on ZF/GAM dynamics

- Opposite ballooning angle
- Density transport
- Momentum transport

Magnetic shaping effect on ITG/TEM instability

- Impact of elongation and triangularity on ITG/TEM turbulence

Control of barrier formation by multi-sources and magnetic shape