Overview of Theory and Simulation Research on Energetic Particle Driven Modes

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Outline

• Burning Plasmas
• Historical development of energetic particle physics: theories and experiments
• Multi-scale couplings: Particle dynamics and characteristic scales, wave-particle interaction
• Energetic particle physics: theories, linear stability and nonlinear simulation codes
• Energetic particle Physics Issues in Burning Plasma: mode stability control, fast ion transport, Helium ash removal, etc.
• Summary
**Fast ion behaviors are critical for burning plasmas**

- Fast ions ($\beta_h \sim \beta_c$) play essential roles in thermal plasma heating and current drive:
  - Fast ions (100 keV - MeVs ) in NBI, N-NBI, ICRH auxiliary heatings
  - 3.5 MeV alphas produced in D-T reaction
- Fast ion driven instabilities (TAEs, RSAEs, RTAEs, EPMs, BAEs, fishbones, etc.) and anomalous fast ion loss have been observed in major magnetic fusion devices.
- Significant fast ion loss can degrade heating and current drive efficiency.
- Lost fast ions tend to localize near outer midplane and can cause localized damage on first wall of fusion reactors.
- In $Q > 5$ burning plasmas, $\alpha$-particles are dominant heating source because $P_\alpha (=W_\alpha/\tau_s) > P_{aux} (=W_{tot}/\tau_E)$. Significant $\alpha$ loss can quench DT burning.
- Alpha heating controls thermal plasma profiles which is essential for global plasma stability and confinement.
Burning Plasmas Physics

- Auxiliary Heating
- D-T fueling
- Current Drive

P(r), n(r), q(r)

- Confinement, Disruption Control
- MHD Stability

- Fusion
- Output

Heating Power: $P_\alpha > P_{aux}$

- Fast Ion Driven Instabilities
- Alpha Transport

Need to understand fast ion physics!
**Fast Ion Physics**

- **First goal** is to insure that super-thermal ions (such as 3.5 MeV alpha particles created by D-T fusion reaction, MeV ion cyclotron wave heated ions, and 100 keV – 1 MeV neutral beam injected ions) are confined well enough to transfer their energy to the thermal plasma, and do not create new plasma instabilities.

- **Second goal** is to understand nonlinear coupling processes involved in fast ion interaction with thermal particles (via profile control) on global stability, confinement, heating and current drive, alpha channeling, burn control, Helium ash removal, thermal instabilities, etc.
Energetic Particle Physics Before 1980s

- Collective effect caused by energetic particles are considered to be unimportant in fusion plasmas

- Slowing-down energy distribution of energetic ions should be stable to velocity-space instabilities in uniform plasma

- MHD modes associated with nonuniform fast ion distributions?
  - Rosenbluth and Rutherford (*PRL* (1975)) considered destabilization of shear Alfven waves (singular modes with frequency in continuous spectrum \( \omega = k_{\parallel}(r_0)V_A(r_0) \)) by interacting with neutral beam injection ions when
    \[
    \omega < \omega_{*b} \quad \text{or} \quad \frac{v_b}{V_A} \frac{\rho_b R}{r_b^2} \frac{mq(r_0)}{nq(r_0) - m} > 1
    \]
  - However, shear Alfven waves are heavily damped by continuum damping effect and was not considered!
  - Shear Alfven continuum modes should be stable.
**Dawning of Energetic Particle Physics Era**

- Ballooning modes are stabilized by trapped electrons and ion FLR effect (Cheng, 1982, Cheng & Gorelenkov, 2004); strong fast ion drive can destabilize resonant kinetic ballooning mode (Cheng et al., 1995).
- Fishbones were first observed in tokamak experiments (McGuire et al., 1983). Fishbones were then explained as internal kink modes destabilized by resonating with fast trapped ion precession drift (Chen, White and Rosenbluth, 1984).
- TAEs (Toroidal Alfven Eigenmodes or Toroidicity-Induced Alfven Eigenmodes) were discovered theoretically and predicted to be easily destabilized by resonating with fast ions (Cheng, Chen and Chance, 1985; Cheng and Chance, 1986; Cheng, Fu and Van Dam, 1988; Fu and Van Dam, 1989).
- First experimental demonstration of TAE destabilization by fast ions in TFTR tokamak was reported in 1991 (Wong et al.).
- Later, experimental observation of TAEs were performed in all major toroidal fusion devices (DIII-D, NSTX, JT-60U, JET, ASDEX, etc.).
- A zoo of Alfven eigenmodes were uncovered from 1990.
Stabilization of Ballooning Mode by Kinetic Effect
Cheng, 1982; Cheng & Gorelenkov, 2004

- Ion FLR effect reduces KBM growth rate, but not the stability boundary
- Trapped electron effect stabilizes KBM, reduces unstable beta domain (trapped electron density $\sim \varepsilon^{1/2}$)

$n=12$ KBM for NSTX with $R/a=1.27$
Fishbones and Fast Ion Loss in PDX

- Excitation of fishbones by fast ions in NBI
- Significant fast ion loss (up to 70%) due to fishbone modes.

McGuire et al., 1983
Trapped Fast Ion Effect on Sawtooth and Fishbones

Chen, White and Rosenbluth (1984) on m/n=1/1 kink mode for q(0) < 1, q(a) > 1

- Sawtooth stabilization (ideal 1/1 kink) for $\beta_{h2} > \beta_h > \beta_{h1}$
- Fishbone mode (resonant 1/1 kink) is destabilized via wave - trapped fast ion precessional drift resonance for $\beta_h > \beta_{h2}$
- n=1 mode perturbation is localized at q=1 surface
- Sawtooth and fishbone can be avoided by operating with q(0) > 1 or with $q_{\text{min}} > 1$ in reversed shear operations.

n=1 kink for PDX equilibrium Cheng (1990)
Theoretical Discovery of TAE modes

• Formation of continuum gap by toroidicity effect (coupling of neighboring poloidal modes)
• Existence of TAE with frequency inside the continuum gap
• TAEs exist for all toroidal n modes (cavity-type modes)
• For each n, there can be many TAEs with different poloidal modes

n=1 TAE computed by NOVA-K code
Alfvén Continuum \((n = 3)\) and TAEs in NSTX

\[<\beta> = 10\%\]

\[<\beta> = 33\%\]

- \(q_0 = 0.7, q_1 = 16\)
- Large continuum gaps due to low aspect ratio even at high \(\beta\).
- Many TAEs with global structure are found and many \(n\) are found.
**High-n TAEs**

- Analytical & numerical solutions of TAEs were obtained for both low-n modes (Cheng & Chance, 1986) and high-n modes (Cheng, Chen, Chance, 1985).
- In a large aspect ratio tokamak with nonuniform q-profile and magnetic field intensity along \( B \), **high-n shear Alfven waves** are described by

\[
\left[ \frac{d^2}{d\theta^2} + \left( \frac{\omega}{\omega_A} \right)^2 \left( 1 - 2\varepsilon \cos \theta \right) - \frac{s^2}{\left( 1 + s^2\theta^2 \right)^2} \right] \Phi = 0
\]

\[\varepsilon = r/R, \quad s = rq'/q, \quad \omega_A = V_A/qR\]
- For zero magnetic shear (\( s = 0 \)), waves are described by the Mathiuequation: waves move in a periodic potential well, similar to electrons moving in a periodic lattice in solid state physics: continuous frequency bands (energy bands) and gaps.
- There is an infinite number of frequency gaps centered at \( \omega \sim j\omega_A/2 \), where \( j = 1, 2, \ldots \).
- The lowest continuum gap is bounded by \( \omega_{\pm}^2 = (1 \pm \varepsilon)\omega_A^2 / 4 \).
High-n TAEs

• For finite magnetic shear \((s \neq 0)\), Toroidal Alfven Eigenmodes (TAEs) with frequency \((\omega \sim \omega_A/2)\) exist in the lowest gap because the periodicity in potential is broken, similar to discrete electron energy states in a periodic lattice due to periodicity breaking by impurity or other effects.

\[
\omega^2 \rightarrow \omega_-^2 \text{ as } s \rightarrow 0 ; \quad \omega^2 \rightarrow \omega_+^2 \text{ as } s \rightarrow \infty
\]

• TAEs are similar to discrete electron energy states in aperiodic lattice due to periodicity breaking by impurity or other effects in solid state physics.
Zoo of Alfvén Eigenmodes in Toroidal Devices

Heidbrink, Phys. Pl. 9 (2002) 2113
Modes that can be Destabilized by Fast Ions

- Kinetic Ballooning Modes (KBM) (Cheng, 1982; Cheng et al., 1995; Cheng & Gorelenkov, 2004)
- Fishbones (resonant internal kink) (McGuire et al., 1983; Chen et al., 1984; Cheng, 1990).
- TAEs, EAEs, NAEs (Cheng et al., 1985; Cheng and Chance, 1986; Cheng et al., 1988; Fu and Van Dam, 1989; Wong et al., 1991, etc.)
- Resonant TAE (RTAE) (Cheng et al. 1995)
- Reverse Shear AE (RSAE) (Kusama et al., 1998; Breizman et al. 2003; Takechi et al., 2005)
- Beta Induced AE (BAE) (Chu et al., 1992; Heidbrink et al., 1992)
- Global AE (GAE) (Goedbloed, 1975; Appert et al., 1982)
- Helicity Induced AE (HAE) (Nakajima et al., 1992)
- Compressional AE (CAE) (Coppi et al., 1986; Gorelenkov and Cheng, 1995)

→ TAEs are generic issue for all toroidal fusion devices!!
→ TAEs are expected to be most serious in fast ion transport!
**TAE Instability**

- Fast ions resonate with TAEs if $V_h \sim V_A$.
  
  For $B = 10T$, $n_e = 2 \times 10^{14} \text{ cm}^{-3}$, $V_A = 10^7 \text{ m/sec}$, $V_\alpha = 1.3 \times 10^7 \text{ m/sec}$. $V_h > 0.5 V_A$ can be satisfied for \(\alpha\)-particles, MeV protons in ICRH operation, and MeV N-NBI Deuterium ions.

- Necessary condition for fast ion drive:
  
  Free energy in fast ion pressure gradient overcomes velocity space damping effect if $nq(V_h/V_A) > (r/R)(L_h/\rho_h)$. 
  
  For large devices (large $L_h/\rho_h$) the unstable spectrum is shifted to medium to high-$n$ modes.

- Sufficient condition for TAE instability:
  
  $\gamma_h$ (fast ion drive) > $\gamma_d$ (thermal plasma damping)

- Multiple TAEs are expected to be robustly unstable in burning plasmas!!
TAE mode structure is resolved using Reflectometry on TFTR DT plasma

- Density eigenmode: n=2
- \( \delta B / B \sim 2 \times 10^{-6} \) No alpha particle loss is observed
TAE density fluctuations in plasma core in TFTR-DT appear before edge magnetic signals

- Reflectometer measures density fluctuations in the plasma core
  - For $\tilde{B}/B \sim 2 \times 10^{-6}$ no alpha particle loss is observed
Bursting TAEs Cause Fast Ion Loss in NSTX

- NSTX shot with $B = 0.434T$, $R = 87$ cm, $a = 63$cm, $P_{NB} = 3.2$MW.
- Single dominant mode being $n=2$ or 3, mode amplitude modulation represents "beating" of multiple modes.
- Bursting TAEs lead to neutron drop and cause $5 - 10\%$ fast ion loss.

(Fredrickson et al., 2003)
Early Observation of Frequency Sweeping seen in Reverse Shear Plasmas: Edge Magnetic Data

- Transition from RSAE (Frequency sweeping) to TAE
Bursting TAEs Observed in JT-60U using NNB

- Slow FS mode (L-RSAE) lasts ~200 ms
- Fast FS mode (H-RSAE) with bursting time of 1 - 5 ms
- Fast ion loss is associated with Bursting TAEs (ALE) with bursting time of 200 - 400 μs and $\delta B/B \sim 10^{-3}$.

(Shinohara et al., 2002)
• After bursting TAE mode occurs neutron emission rate \( (S_n) \) drops and enhanced fast neutral fluxes \( (G) \) are observed.

• Bursting TAE modes cause enhanced transport of energetic ions via wave-particle resonant interaction.

(Shinohara et al., 2002)
Change of Neutron Profile by Bursting TAEs

- After bursting TAE modes peripheral signals \( (r/a>0.48) \) increase while center signals \( (r/a<0.34) \) decrease.

\[ \Rightarrow \] Bursting TAEs causes global redistribution of energetic ions.

Ishikawa et al. 2005
Fast Ion Transport by Bursting TAEs

E39672 at 4.51sec

Ishikawa et al. 2005
Reversed Shear Alfvén Eigenmode (RSAE)

- RSAEs are cylindrical shear Alfven modes in reversed shear plasmas
- RSAEs are described by ideal MHD theory

\[ n = 1 \text{ continuum} \]
MHD Theory of RSAE

- RSAE frequency: $\omega \sim (m - nq_{\min})V_A / q_{\min}R$
- RSAEs are modes localized at $q_{\min}$
- Finite $\beta$ causes non-zero RSAE frequency due to poloidal mode coupling at $q_{\min}$

Kramer et al., 2004
NOVA-K Calculation of RSAEs in JET

- Good agreement between RSAE theory and observed frequencies
- Theory predicts frequency gaps where no modes exist
- RSAE stability agrees with mode observation
Will TAEs Be Unstable in ITER?

TAEs are expected to be unstable in ITER!

What is the effect on $\alpha$ transport?

- TAEs are expected to be unstable in ITER!
- What is the effect on $\alpha$ transport?
• **Disparate scales in plasmas**: traditionally global-scale and long time behavior phenomena are studied using MHD or multi-fluid models, while small spatial and short temporal scale phenomena are described by kinetic theories.

• **Multiscale coupling**: small temporal and spatial scale particle kinetic physics couple with long temporal & large spatial scale phenomena.

• **Energetic particle physics are multiscale coupling phenomena**: global instabilities are driven by fast ions via wave-particle interaction and cause serious fast ion loss in tokamaks.

• **Thermal particle kinetic physics**: thermal particle kinetic effects are as important as fast ion’s in determining global phenomena in burning plasmas.
Characteristic Scales in Fusion Plasmas

- Characteristic scales of particle dynamics and low-frequency (ω, k) perturbations:

For B = 10 T, Te,i = 10 keV, εh > 1 MeV, L_B, L_p \geq O(1 \text{ m}),
then we have \( \rho_i \sim 0.1 \text{ cm} \), \( v_i \sim 10^8 \text{ cm/s} \),
\( \omega_{ci} \sim 10^9 \text{ sec}^{-1} \), \( \omega_{bi} \sim 10^6 \text{ sec}^{-1} \), \( \omega_{di} \sim 10^5 \text{ sec}^{-1} \)

-- temporal scale ordering:

\[ \omega_{ci} > \omega_{be}, \omega_{bh} > \omega_{bi} \sim \omega_{i,e} \sim \omega_{d,e} \]

-- spatial scale ordering:

\[ \Delta_{bi} > \rho_h > \rho_i \sim c/\omega_{pi} > \rho_e \]

- To describe low-frequency (ω, k) phenomena, MHD model is a good approximation only if (a) \( \omega_{ci} \gg \omega \gg \omega_t, \omega_b, \omega_d \) and (b) \( k \rho_i \ll O(1) \) are satisfied for all particle species that have significant contributions in density, momentum and pressure.
Multi-Scale Coupling

- **Multi-spatial scale coupling**
  - Finite ion Larmor radius effect \((k_\perp \rho_i \sim O(1))\):
    1. \(\vec{V}_\perp \neq \vec{V}_\perp e \sim \vec{E} \times \vec{B} / B^2\)
    2. parallel electric field \(E_\parallel\)
    3. ion gyroviscosity
  - Finite banana orbit width effect \((k_\perp \Delta_b \sim O(1))\)
  - Small magnetic field scale length \((L_B \sim \rho_i)\): particle magnetic moment is not conserved

- **Multi-temporal scale coupling** \((\omega_t > \omega_b > \omega_d)\)
  - \(\omega - \omega_d - \omega_{b,t} = 0\), transit/bounce resonances will cause wave energy dissipation or growth (fast ion driven TAE instabilities)
  - \(\omega < \omega_b\), trapped particles will respond to an bounce orbit-averaged field (trapped electron stabilization effects on kinetic ballooning instabilities)
  - \(\omega \sim \omega_d\), wave-particle drift resonance effects are important for energy dissipation or release (fast ion driven fishbones)
  - \(\omega \ll \omega_d\), particle magnetic drift motion dominates over \(E \times B\) drift (sawtooth stabilization by fast ions)

→ Particle kinetic effects are essential in burning plasmas.
Energetic Particle Physics Models

- **Vlasov-Boltzmann models**: PIC or Vlasov phase space fluid models with full particle dynamics
- **Gyrokinetic model**: dynamics of particle gyration is averaged over the gyration temporal and spatial scales; PIC or phase space fluid simulation. Electrons are treated with fluid-kinetic model.
- **Kinetic-MHD model**: hybrid model with fast ion kinetic physics coupled to MHD fluid model via pressure coupling; usually, but not limited to, gyrokinetic model is employed to describe particle dynamics ($E_{\parallel} = 0, n_h \ll n_c$)
- **Kinetic-fluid model**: hybrid model with kinetic physics of all particle species coupled to single-fluid model via pressure tensor coupling

The goal is to develop a nonlinear simulation code that treats kinetic physics of both thermal and fast particles for studying long time behaviors of global burning plasma phenomena.
Considerations of Numerical Modeling

- **Physics consideration**
  -- energetic dynamics
  -- thermal ion & electron dynamics? Time scales of particle motion & high frequency waves?

- **Numerical methods**
  -- time step (dictated by both numerical method)
  -- coordinate system, spatial grid or mode expansion
  -- finite difference methods

- **Computational error**
  -- accumulated numerical error from finite difference methods

- **Analysis of numerical output**
  -- amount of information, manpower
  -- physics understanding

- **Computer requirement**
  -- CPU speed, memory, ease of use, etc.
Linear Stability Codes

- **NOVA-K** (Cheng (1992))
  -- Based on kinetic-MHD model; include ion FLR in MHD part, fast ion contributions are treated either perturbatively or non-perturbatively

- **HINST** (Gorelenkov et al. (1998))
  -- High-n TAE stability code

- **CASTOR-K** (Borba & Kerner (1999))
  -- Based on kinetic-MHD model; include ion FLR in MHD part, fast ion contributions are treated perturbatively

- **LIGKA** (Lauber et al. (2005; 2007))
  -- All species are treated with gyrokinetic equations with small FLR expansion ($k\rho << 1$)

- **GYGLES** (Mishchenko et al. (2008, 2009))
  -- Time dependent linear gyrokinetic PIC model

- **GEM** (linear version) Chen et al. (2010)
  -- Time dependent linear gyrokinetic PIC model

- **TASK/WM**
  -- Wave equations with damping effect
  -- Fukuyama & Tohnai (1997); Fukuyama & Yokota (1999)
Nonlinear Simulation Codes

• **Kinetic-MHD codes**
  -- Bulk plasma is treated by MHD model and fast ions are treated by kinetic models
  -- M3D-K code: Park et al.(1992) with gyrokinetic PIC model;
  -- HYM code: Belova (2004) with full ion PIC model;
  -- MEGA code: (Todo et al. 1995; Wang & Todo, 2011) with $\delta f$ drift kinetic PIC model

• **TAEFL Code** (Spong et al. (1992, 1994))
  -- Fluid formulation with Landau damping/growth closures

• **Gyrokinetic PIC Toroidal Codes**
  -- Thermal and fast ions are treated by low-frequency gyrokinetic PIC model, electrons are treated by fluid-kinetic model.
  -- Lee (1987); Lin et al. (1998); Holod et al. (2009, 2013); Lang et al. (2009)

• **GYRO-fluid Code** (Candy and Waltz (2003))
  -- all species are treated by solving $\delta f$ gyrokinetic equations in phase space; all species have Maxwellian velocity distributions; high-n ballooning formalism is used.

• **Kinetic-Fluid codes**
  -- All particles are treated by gyrokinetic models, but couple to single-fluid
  -- Cheng & Johnson (1999); Nishimura & Cheng (2010)
Some Considerations:

- Mode stability can be achieved by enhancing damping and/or reducing fast ion drive.
- Both fast ion drive and thermal particle damping depend on profiles of $q(r)$, $n(r)$, and $T(r)$.
- Control $q(r)$ and $n(r)$ profiles so that the radial structure of continuum gap does not line up across minor radius and continuum damping is enhanced.
- Increase plasma $\beta$ near ballooning limit so that mode frequency will move into continuum and TAE can be stabilized by continuum damping.
- Rotate plasma to modify continuum gap structure so that modes can be stabilized by thermal particle & continuum damping.
Fast Ion Loss due to Energetic Particle Modes

- TAEs with a wide spectrum of $n$ are expected to be unstable in wide radial domain in burning plasmas. Other EP driven modes are more localized in radial domain.

- TAEs induces particle drift orbit islands in phase space. Fast ions in drift orbit islands are lost to the first wall if drift orbit islands overlap with the prompt loss domain with a transient loss rate $\sim (\delta B_r/B)^{1/2}$.

- When multiple TAEs are excited, stochastic diffusion loss can occur if multiple drift orbit islands overlap and if particles can diffuse stochastically into the prompt loss domain.

- Stochastic loss rate $\sim (\delta B_r/B)^2$ if TAE amplitude exceeds orbit stochastic threshold of $\delta B_r/B \sim 10^{-3}$ for single TAEs and $\sim 10^{-4}$ for multiple TAEs.

- Alpha loss due to multiple TAEs in burning plasmas could be significant.
Alpha Particle Interactions with RF Waves

- Removal of helium ash (~ a few hundred keV)
  - Convective bucket transport by externally driven RF waves with frequency chirping
- Alpha current drive
- Alpha channeling – transfer alpha energy to thermal ions directly
- Plasma flow generation and control using fast ions
- Burning plasma devices such as ITER can explore these ideas!
Summary

- **Energetic particle physics** is an integral part of burning plasma physics – including heating, current drive, profile control, burning physics integration.
- Fast ion instabilities and transport are among the most critical issues of energetic particle physics at present stage.
- Many fast ion driven instabilities have been identified. **TAEs are potentially most serious in causing fast ion transport.**
- Close interaction between theory/simulation and experiment has yielded significant progress in understanding energetic particle physics.
- **Study of integrated burning physics** requires long-time simulation at least for several energy confinement times and should be performed in the near future.
- **Burning physics integration and profile control** cant be studied with ITER burning plasmas.
Toward Long-Time Scale Simulation of Burning Plasma Physics

Collaboration between Experimental and Theory Groups

SUPER-CODES: K-Fluid Simulation of Wave-Particle Interaction Physics & Burning Plasma Physics

NOVA-KF RF Physics

NOVA-2: Non-perturbative Kinetic Effects

HINST Kinetic Thermal Particle Physics

NOVA-K Low- to Medium-n Modes

HINST High-n Modes

ORBIT

K-Fluid Code: Kinetic Fast & Thermal Particles

ORBIT: Fast ion -RF Interactions

M3D-K HYM ORBIT

Advances in Computing Power and Theoretical Understanding

Kinetic-MHD Models Kinetic-Fluid