Two-fluid/FLR Effects on Kelvin-Helmholtz Instability

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Background

- The flow in plasmas in magnetic confinement fusion devices can play important roles.
- Shear flow can improve plasma confinement.
  ↔ It can cause Kelvin-Helmholtz (KH) instability. [1][2]
- (Single-fluid) magnetohydrodynamics (MHD) model omits small scale effects.
  - Ion inertia (two-fluid) effect [3]
  - Finite Larmor radius (FLR) effect [4]

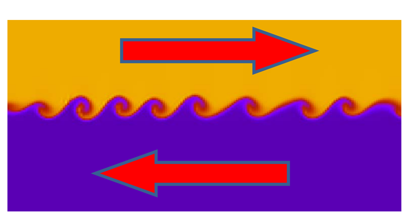
Small scale effects are important when the shear is strong. (E.g. the edge region of H-mode tokamaks)

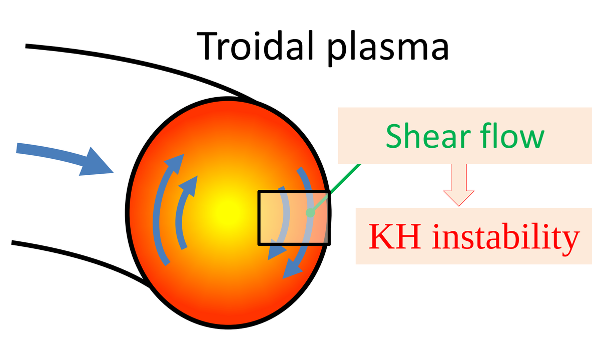
→ Extended MHD model

= (single-fluid) MHD + two-fluid terms + FLR terms (gyroviscosity)

Kelvin-Helmholtz Instability

- A magnetohydorodinamic (MHD) instability driven by shear flow
- In troidal plasma in magnetic confinement fusion devices, sheared poloidal flow can cause KH instability


Typical KH vortices



Model and Method

- Extended MHD equations

$$\frac{\partial}{\partial t}(\rho) = -\nabla\cdot[\rho\boldsymbol{v}]$$

$$\frac{\partial}{\partial t}(\rho\boldsymbol{v}) = -\nabla\cdot\left[\rho\boldsymbol{v}\boldsymbol{v} + \mathbf{I}\left(p+\frac{B^2}{2}\right) - \boldsymbol{B}\boldsymbol{B} + \delta\boldsymbol{\Pi_i}\right]$$

$$\frac{\partial}{\partial t}\left(\frac{\rho v^2}{2} + \frac{p}{\gamma-1}\right) = -\nabla\cdot\left[\boldsymbol{v}\left(\frac{\rho v^2}{2} + \frac{\gamma p}{\gamma-1}\right) - \boldsymbol{v}\cdot\delta\boldsymbol{\Pi_i}\right]$$

$$\frac{\partial}{\partial t}(\boldsymbol{B}) = -\nabla\cdot[\boldsymbol{v}\boldsymbol{B} - \boldsymbol{B}\boldsymbol{v}] - \nabla\times\left[\frac{\varepsilon}{\rho}(\boldsymbol{J}\times\boldsymbol{B} - \nabla p_e)\right]$$

Gyroviscosity

Two-fluid effect terms

$p = p_i + p_e$: Total Pressure

γ : Adiabatic index

ε : Hall parameter

δ : Gyroviscosity parameter

Essentially,

$\delta = \varepsilon = \frac{d_i}{L}$ — d_i : Ion inertia length, L : Scale length

however, different values are set to track the FLR effect and the two-fluid effect, respectively.

- In a 2D slab

$$v_z = 0, \quad \frac{\partial}{\partial z} = 0$$

Gyroviscous tensor

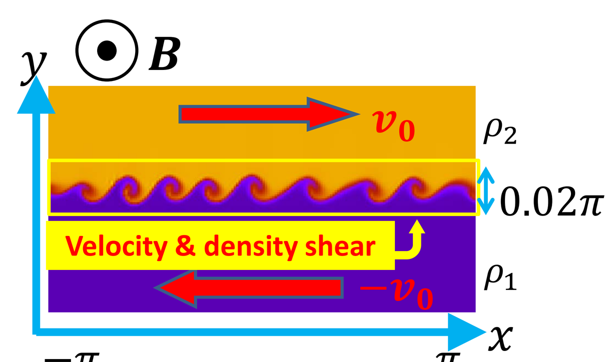
$$(\boldsymbol{\Pi_i})_{xx} = -(\boldsymbol{\Pi_i})_{yy} = -\frac{p_i}{2B}\left(\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y}\right)$$

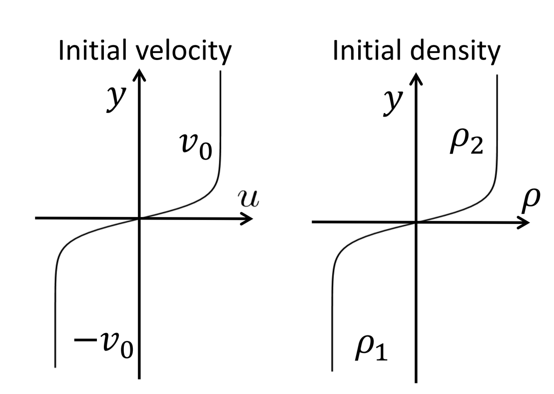
$$(\boldsymbol{\Pi_i})_{xy} = (\boldsymbol{\Pi_i})_{yx} = \frac{p_i}{2B}\left(\frac{\partial v_x}{\partial x} - \frac{\partial v_y}{\partial y}\right)$$

- Numerical method
  - 4th-order central difference for spatial derivatives
  - 4th-order Runge-Kutta-Gill (RKG) scheme for time evolution
  - Resolution : 512 X 2048

Initial Condition

$\boldsymbol{B} = B_0\hat{\boldsymbol{z}}$, $B_0 = 1.0$, $\beta = \frac{p_{y=\infty}}{B_0^2/2}$





Initial equilibrium

$$\frac{\partial}{\partial y}\left(p + \frac{B_0^2}{2} + \delta\frac{p_i}{B_0}\frac{\partial v_x}{\partial y}\right) = 0$$

Boundary condition

X-direction : periodic

Y-direction : $\frac{\partial}{\partial y} = 0$

$$v_x = v_0\tanh\left(\frac{y}{d}\right) \quad d = 0.01\pi$$

$v_0 = 0.1$ or -0.1

$$\rho = \frac{\rho_1+\rho_2}{2} + \frac{\rho_2-\rho_1}{2}\tanh\left(\frac{y}{d}\right)$$

$\rho_1 = 1.0 \quad \rho_2 = 4.0$

$p_i = p_e$

With random perturbation

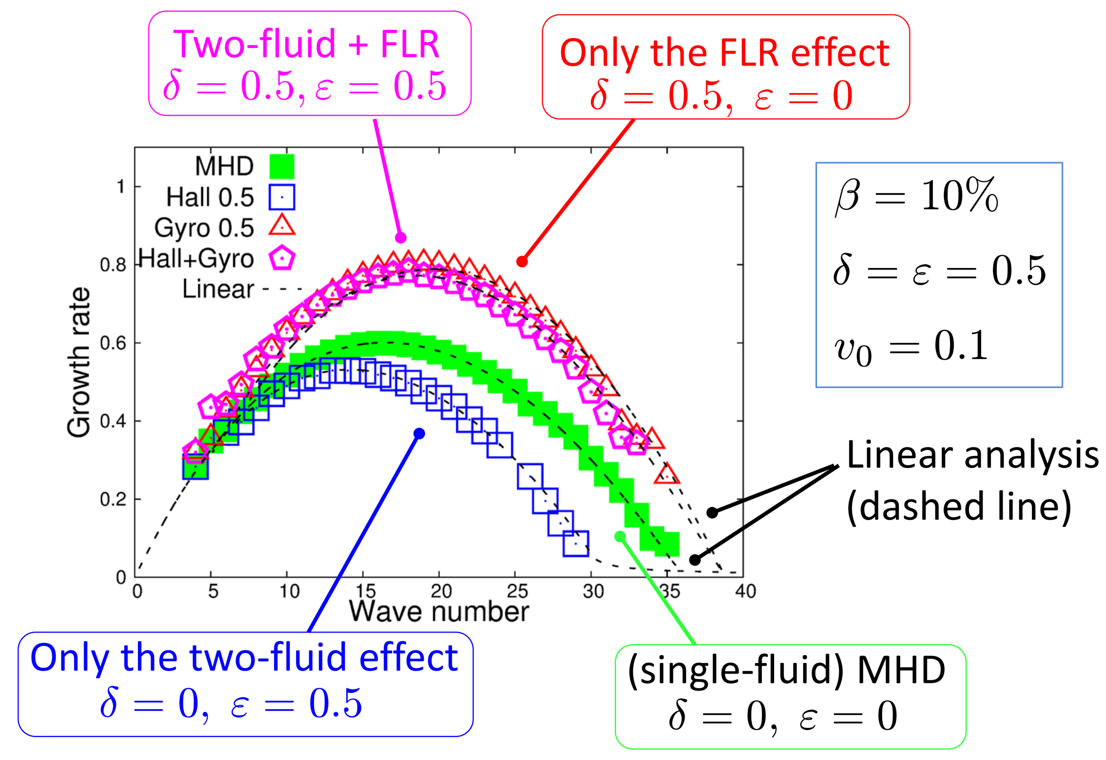
Result and Analysis

Validation

A comparison of growth rates between the simulation and the linear analysis

[A. Ito, JPS 2013 annual meeting:26aEA-5]

- The simulation is consistent with the linear analysis.



The two-fluid effect and the FLR effect

Typical fusion plasma parameters: $B \sim 1\,\mathrm{T}$, $n \sim 10^{19}\,\mathrm{m}^{-3}$, $T \sim 10\,\mathrm{kev}$ ⇒ $d_i \sim 7.2\,\mathrm{cm}$, $r_L \sim 1.4\,\mathrm{cm}$, $\beta \sim 0.04$

n : Number density

T : Temperature

r_L : Larmor radius

In the case $L \sim 30\,\mathrm{cm}$ ⇒ $\delta = \varepsilon \sim 0.24$

Parameters for the simulation

- Only the two-fluid effect: $\delta = 0, \varepsilon = 0.25$
- Only the FLR effect: $\delta = 0.25, \varepsilon = 0$
- (single-fluid) MHD: $\delta = 0, \varepsilon = 0$
- Two-fluid + FLR: $\delta = 0.25, \varepsilon = 0.25$

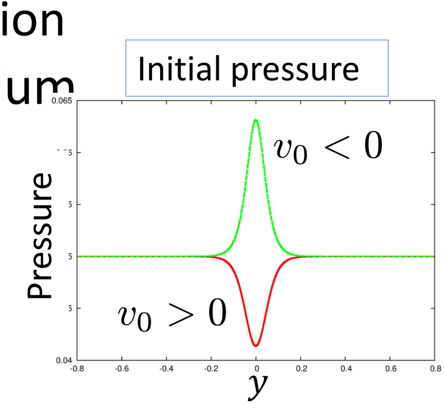
- Influence on growth rates increases with β.
- The two-fluid effect reduces growth rates in most cases.
- The FLR effect enhances growth rates.
- With the both effects, the influence on growth rates is not a simple addition of the two effects.

→ Growth rates are rather enhanced.

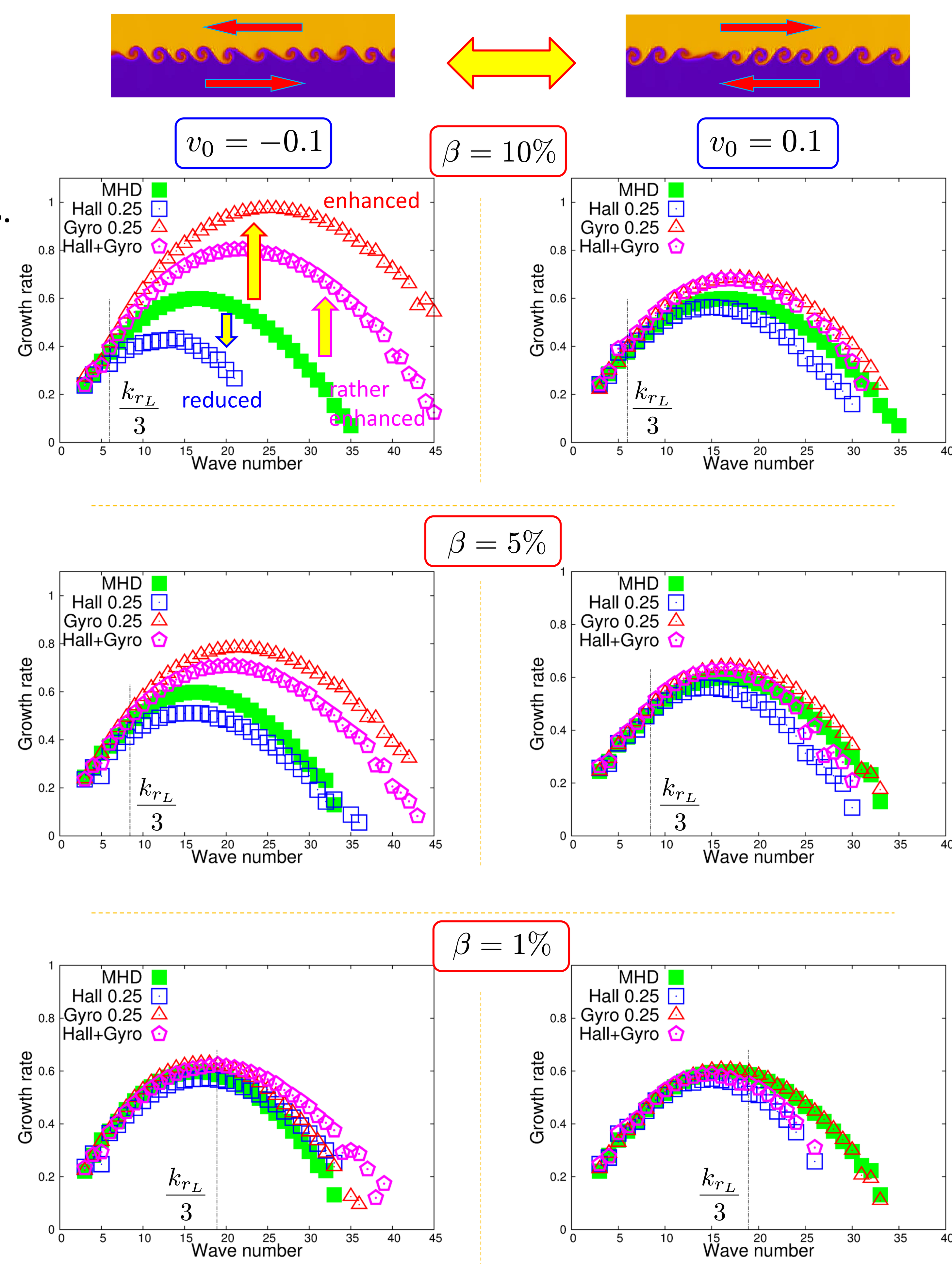
The dispersion relation of KH instability can be in a complicated form of the terms from the two effects.

1. The both effects have asymmetry with respect to the velocity v_0.
   - Density gradient → Diamagnetic current
   - Background magnetic field → Ion gyration
   - Initial equilibrium

Asymmetry in pressure profile



- Wave numbers affected by the FLR effect vary with β.



Summary

- The simulation is consistent with the linear analysis.
- The two-fluid effect reduces growth rates of KH modes in the scale smaller than the ion inertia length.
- The FLR effect enhances growth rates of KH modes in the scale smaller than three times the Larmor radius.

[1]Y. Idomura, M. Wakatani and S. Tokuda, Phys. Plasmas 7, 3551 (2000)
[2] R. Numata, R. Ball and R.L. Dewar, Phys. Plasmas 14, 102312 (2007)
[3]J.D. Huba, Phys. Rev. Lett. 72, 2033 (1994)
[4] J.D. Huba, Geophys. Res. Lett. 23, 2907 (1996)

Future plan

- Higher resolution
  - Parallel computing with MPI
  - Implementation of AMR module
- Mechanism of two-fluid effect and FLR effect
- Combined Rayleigh-Taylor/Kelvin-Helmholtz instability ⇒ ELMs in tokamaks
- 3D torus simulation